MATHEMATICS

CLASS XI

(Under AHSEC Curriculum)

Content Developed by-

Dr. Dipak Sarma
&
Dr. Parag Kumar Deb
Associate Professor
Dept. of Mathematics
Cotton College, Assam

Dr. Anjana Bhattacharyya
Associate Professor
Dept. of Mathematics
B. Borooah. College, Assam

\

Assam Electronics Development Corporation Limited
(AMTRON)
(A Government of Assam Undertaking)

Name of the textbook

MATHEMATICS

Name of the Chapter

Statistics

(Chapter 15)

CONTENTS

	Page No.
1. Study Materials	15.1-15.22
2. Exercise 15	15.23-15.26

CHAPTER
15

STATISTICS

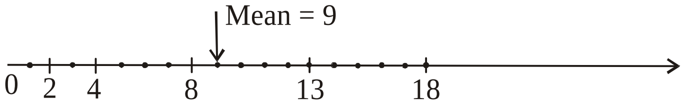
Statistics is the science which deals with the collection, tabulation, analysis and interpretation of numerical data to throw light on any sphere of enquiry.

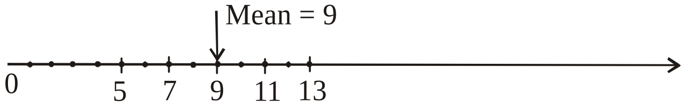
Dispersion : The variation or scattering or deviation of the different values of a variable from their average is known as dispersion.

Consider the following two series of values or observations :

(i) 2, 4, 8, 13, 18 (ii) 5, 7, 9, 11, 13

We plot the values of the two series on number line.

(i)


(ii)


Mean of (i) is $\frac{\sum x_i}{n} = \frac{2+4+8+13+18}{5} = \frac{45}{5} = 9$

Mean of (ii) is $\frac{\sum x_i}{n} = \frac{5+7+9+11+13}{5} = \frac{45}{5} = 9$

From the two graphs, we see that the values 2, 4, 8, 13, 18 of (i) are more dispersed from the mean 9, than the values 5, 7, 9, 11, 13 of (ii) through the two series of values have the same mean 9.

The common measures of dispersion are–

(i) Range (ii) Quartile deviation (iii) Mean deviation (iv) Standard deviation

Here, we shall study all the measures of dispersion except Quartile Deviation.

For Simple Series :

$$Mean\left(\bar{x}\right) = \frac{\sum x_i}{n}, \quad \text{where } n = \text{number of values.}$$

Median : After arranging the values in ascending order of magnitude, we get

$$\text{Median} = \left(\frac{n+1}{2}\right)th \text{ value, if n is odd.}$$

$$\text{Or} \quad = \text{mean of } \left(\frac{n}{2}\right)th \text{ and } \left(\frac{n}{2}+1\right)th \text{ values, if n is even.}$$

Example-1 : Find the mean and median of the following observations :

20, 24, 29, 35, 43, 34, 25.

Solution : Mean $\left(\bar{x}\right) = \frac{20+24+29+35+43+34+25}{7} \qquad \left[\frac{\sum x_i}{n}\right]$

$$= \frac{210}{7} = 30$$

To find the median, we first arrange the given observations in ascending order of magnitude :

20, 24, 25, 29, 34, 35, 43

Here n = number of observations = 7, which is odd

$\therefore$ Median = $\left(\frac{n}{2}+1\right)th$ i.e. $\left(\frac{7+1}{2}\right)th$ i.e. 4th observation

= 29.

Example-2 : Find the median of the following observations :

20, 25, 17, 18, 8, 15, 22, 11, 9, 14.

Solution : Arranging the data in ascending order of magnitude we have :

8, 9, 11, 14, 15, 17, 18, 20, 22, 25

Here n = 10 which is even.

Median = Mean of $\left(\frac{n}{2}\right)th$ and $\left(\frac{n}{2}+1\right)th$ observations i.e. mean of 5th and 6th observations

$$=\frac{15+17}{2}=16$$

Range : It is the difference between the largest and smallest values of a variable.

Thus, Range = L – S, where L = Largest value, S = Smallest value

Illustration : Range of the values 10, 20, 16, 30, 10, 80, 60, 90, 70 is 90 – 10 = 80

Mean Deviation : The mean deviation about 'a' denoted by M.D (a) is defined as the mean of the absolute values of the deviations of the observation from 'a'. Thus

$$\text{M.D(a)} = \frac{\text{Sum of the absolute values of deviations from 'a'}}{\text{Number of deviations}}.$$

Mean deviation for ungrouped data : Let, n observations be x_1, x_2, ...x_n, then mean deviation about 'a' is given by

$$\text{M.D (a)} = \frac{\sum_{i=1}^{n}|x_i - a|}{n}$$

Similarly

$$\text{M.D }(\overline{x}) = \frac{\sum_{i=1}^{n}|x_i - \overline{x}|}{n}, \text{ where } \overline{x} = \text{Mean}$$

$$\text{M.D }(M) = \frac{\sum_{i=1}^{n}|x_i - M|}{n}, \text{ where M=Median}$$

Example-3 : Find the mean deviation about the mean for the following data :

38, 70, 48, 40, 42, 55, 63, 46, 54, 44

Solution : $Mean\left(\overline{x}\right) = \frac{38+70+48+40+42+55+63+46+54+44}{10}$

$$=\frac{500}{10}=50$$

The respective values of deviations from mean i.e. $x_i - \overline{x}$ are 38–50, 70–50, 438–50, 3840–50, 42–50, 55–50, 63–50, 46–50, 54–50, 44–50, 38–50 i.e., –12, 20, –2, –10, –8, 5, 13, –4, 4, –6

The respective absolute values of the deviations from mean i.e. $|x_i - \bar{x}|$ *are* 12, 20, 2, 10, 8, 5, 13, 4, 4, 6.

$$\therefore \quad M.D(\bar{x}) = \frac{\sum |x_i - \bar{x}|}{n} = \frac{12+20+2+10+8+5+13+4+4+6}{10}$$

$$= \frac{84}{10} = 8.4$$

Example-4 : The following data gives the weights (in grams) of 15 oranges picked up from a basket :

106, 107, 76, 82, 109, 107, 115, 92, 127, 95, 123, 126, 76, 92, 86

Calculate the mean deviation about the median.

Solution : Arranging the weights of the oranges in ascending order, we get

76, 76, 82, 86, 92, 92, 95, 106, 107, 107, 109, 115, 123, 126, 127

Here n = 15, which is odd

$\therefore$ M = Median = $\left(\frac{n}{2}+1\right)th$ observation = $\left(\frac{15+1}{2}\right)th$ observation = 8th observation = 106

Weights (in gms)	x_i **– M**	$\lvert x_i$ **– M**$\rvert$
76	–30	30
76	–30	30
82	–24	24
86	–20	20
92	–14	14
92	–14	14
95	–11	11
106	0	0
107	1	1
107	1	1
109	3	3
115	9	9
123	17	17
126	20	20
127	21	21
		$\Sigma\lvert x_i – M\rvert = 215$

$$\therefore \quad M.D(\bar{x}) = \frac{\sum |x_i - M|}{n} = \frac{215}{10} = \frac{43}{3} = 14.3$$

Example-5 : Calculate the mean deviation about the median for the daily wages of 12 labours.

Wages (in rupees) : 50, 48, 45, 60, 50, 46, 48, 50, 45, 70, 65, 47

Solution : Arranging the wages in ascending order we get

45, 45, 46, 47, 48, 50, 50, 50, 60, 65, 70

Here n = 12, which is even.

$\therefore$ M = Median = Mean of $\left(\frac{n}{2}\right)th$ and observation = $\left(\frac{n}{2}+1\right)th$ observations

= Mean of 6th and 7th observations = $\frac{48+50}{2} = 49$

Wages	x_i – M	$\|x_i$ – M$\|$
45	–4	4
45	–4	4
46	–3	3
47	–2	2
48	–1	1
48	–1	1
50	1	1
50	1	1
50	1	1
60	11	11
65	16	16
70	21	21
		$\Sigma\|x_i–M\|=66$

Mean deviation about Median $=\frac{\sum\left|x_i-M\right|}{n}=\frac{66}{12}=\frac{11}{2}=5.5$

Example-6 : The mean of 6, 8, 5, 7, a and 4 is 7. Find the mean deviation about median of these observations.
Solution : Here n = 6

By given conditions, $7=\frac{6+8+5+7+a+4}{6}$

⇒ 42 = 30 + a

⇒ a = 12

The observations, when arranged in ascending order are

4, 5, 6, 7, 8, 12

∴ M = Median = Mean of $\left(\frac{6}{2}\right)th$ and $\left(\frac{6}{2}+1\right)th$ observations

= Mean of 3th and 4th observations = $\frac{6+7}{2}=6.5$

x_i	x_i – M	$\|x_i$ – M$\|$
4	– 2.5	2.5
5	– 1.5	1.5
6	– 0.6	0.6
7	0.5	0.5
8	1.5	1.5
12	5.5	5.5
		$\Sigma\|x_i–M\|=12$

∴ $M.D=\frac{\sum\left|x_i-M\right|}{n}=\frac{12}{6}=2$

Mean deviation for grouped data :

Data can be grouped into two ways :

(i) Discrete frequency distribution

(ii) Continuous frequency distribution

(i) Discrete frequency distribution : Let the frequency distribution be

x : x_1 , x_2 , x_3 , ...x_n
f : f_1 , f_2 , f_3 , ...f_n

Then $$\text{M.D}(\bar{x}) = \frac{\sum_{i=1}^{n}|x_i - \bar{x}|}{\sum_{i=1}^{n} f_i} = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$$

where $N = \sum_{i=1}^{n} f_i$ and $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$

$$\text{M.D}(M) = \frac{\sum_{i=1}^{n} f_i |x_i - M|}{N}$$

where M = Median = the value of x corresponding to cumulative frequency is equal to or just greater than $\frac{N}{2}$.

(ii) Continuous frequency distribution :

Let the continuous frequency distribution be :

Class-intervals :	$l_1 – l_2$	$l_2 – l_3$	....	$l_{n-1} – l_n$	$l_n – l_{n+1}$
Frequency :	f_1	f_2		f_{n-1}	f_n

$$\text{M.D}(\bar{x}) = \frac{1}{N} \sum_{i=1}^{n} f_i |x_i - \bar{x}|$$

where $\bar{x} = \frac{\sum_{i=1}^{n} f_i |x_i - \bar{x}|}{N}$, x_i is mid-value of the ith class-interval

$$\text{M.D}(M) = \frac{1}{N} \sum_{i=1}^{n} f_i |x_i - M|$$

where $M = l + \frac{\left(\frac{N}{2} - C\right)}{f} \times n,$

l = lower limit of the median class
f = frequency of the median class
n = length of the median class
C = cumulative frequency of the class just before of the median class.

Note : (i) Median class is the class whose cumulative frequency is just greater than or equal to $\frac{N}{2}$.

(ii) $Mean, \bar{x} = a + \frac{\sum f_i d_i}{N} \times n$, where $d_i = \frac{x_i - a}{n}$ and a is the assumed mean.

Example-7 : Find the mean deviation about the mean for the following data :

x_i	10	30	50	70	90
f_i	4	24	28	16	8

Solution : We make the following table :

x_i	f_i	$f_i x_i$	$\lvert x_i - \bar{x} \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
10	4	40	40	160
30	24	720	20	480
50	28	1400	0	0
70	16	1120	20	320
90	8	720	40	320
	Σf_i=80	$\Sigma f_i x_i$=4000		$\sum f_i \lvert x_i - \bar{x} \rvert = 1280$

N = Σf_i=80

$$\bar{x} = \frac{\sum f_i x_i}{N} = \frac{4000}{80} = 50$$

$$M.D(\bar{x}) = \frac{\sum f_i \lvert x_i - \bar{x} \rvert}{N} = \frac{1280}{80} = 16$$

Example-8 : Find the mean deviation about the median for the following data :

x_i	5	7	9	10	12	15
f_i	8	6	2	2	2	6

Solution :

x_i	f_i	c.f	$\lvert x_i - M \rvert$	$f_i \lvert x_i - \bar{x} \rvert$
5	8	8	2	16
7	6	14	0	0
9	2	16	2	4
10	2	18	3	6
12	2	20	5	10
15	6	26	8	48
	Σf_i=26			$\sum f_i \lvert x_i - \bar{x} \rvert = 84$

Here N = Σf_i = 26, which is even.

Median is the mean of $\frac{N}{2}$th and $\left(\frac{N}{2}+1\right)$th observations i.e. 13th and 14th observations. Both of these observations lie in the cumulative frequency 14 for which the corresponding observation is 7.

$$\therefore \quad \text{Median} = \frac{\text{13th observation+14th observation}}{2} = \frac{7+7}{2} = 7$$

$$\therefore \quad \text{M.D(M)} = \frac{\sum f_i \lvert x_i - M \rvert}{N} = \frac{84}{26} = 3.23$$

Example-9 : Find the mean deviation about the median for the following set of observations :

x_i	15	21	27	30	35
f_i	3	5	6	7	8

Solution :

x_i	f_i	c.f	$\lvert x_i - M \rvert$	$f_i \lvert x_i - M \rvert$
15	3	3	15	45
21	5	8	9	45
27	6	14	3	18
30	7	21	0	0
35	8	29	5	40
	Σf_i=29			$\sum f_i \lvert x_i - M \rvert = 148$

Here N = Σf_i = 29, which is odd.

∴ Median $= \left(\frac{29+1}{2}\right)$th observation = 15th observation and this observation lie in the cumulative frequency 21. The corresponding observation is 30.

∴ Median = 30

∴ M.D(M) $= \frac{\sum f_i \lvert x_i - M \rvert}{N} = \frac{148}{29} = 5.1$

Example-10 : Calculate the mean deviation about the mean for the following data :

Income per day	0 – 100	100 – 200	200 – 300	300 – 400	400 – 500	500 – 600	600 – 700	700 – 800
Number of person	4	8	9	10	7	5	4	3

Solution :

Income per day	***Number of persons*** **(f_i)**	***Mid points*** **(x_i)**	$f_i x_i$	$\lvert x_i - \overline{x} \rvert$	$f_i \lvert x_i - \overline{x} \rvert$
0–100	4	50	200	308	1231
100–200	8	150	1200	208	1664
200–300	9	200	2250	108	972
300–400	10	350	3500	8	80
400–500	7	450	3150	92	644
500–600	5	550	2750	192	960
600–700	4	650	2600	292	1168
700–800	3	750	2250	392	1176
	Σf_i=50		$\Sigma f_i x_i$=17900		$\sum f_i \lvert x_i - M \rvert = 7896$

∴ Mean $(\overline{x}) = \frac{\sum f_i x_i}{\sum f_i} = \frac{17900}{50} = 358$

$$M.D\lvert \overline{x} \rvert = \frac{\sum f_i \lvert x_i - \overline{x} \rvert}{\sum f_i} = \frac{7896}{50} = 151.92$$

Example-11 : Find the mean deviation about the median for the following data :

Marks	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60
Number of girls	8	10	10	16	4	2

Solution : We construct the following table :

Marks	**Number of girls (f_i)**	**Cumulative frequency (c.f.)**	**Mid points (x_i)**	**$\|x_i – M\|$**	**$f_i\|x_i – M\|$**
0–10	8	8	5	22	176
10–20	10	18	15	12	120
20–30	10	28	25	2	20
30–40	16	44	35	8	128
40–50	4	48	45	18	72
50–60	2	50	55	28	56
	Σf_i=50				$\sum f_i \| x_i - M \| = 572$

Hence, N = Σf_i=50. The class containing $= \frac{N}{2} = \frac{50}{2} = 25$ item is 20–30, which is the Median Class

Median $= l + \frac{\left(\frac{N}{2} - C\right)}{f} \times h$ [c = 18, h = 10, f = 19]

$$= 20 + \frac{25-18}{10} \times 10 = 27$$

∴ M.D(M) $= \frac{\sum f_i \left|x_i - M\right|}{\sum f_i}$

$$= \frac{572}{50} = 11.44$$

Example-12 : Calculate the mean deviation about the median for the age distribution of 100 persons given below.

Age	16 – 20	21 – 25	26 – 30	31 – 35	36 – 40	41 – 45	46 – 50	51 – 55
Number of persons	5	6	12	14	26	12	16	9

Solution : The given frequency distribution is discontinuous, to convert it into continuous frequency distribution, adjustment factor is $\frac{21-20}{2} = 0.5$

So we subtract 0.5 from the lower limit and add 0.5 to the upper limit of each class.

We construct the following table :

Age	Number of persons (f_i)	c.f	Mid points (x_i)	$\|x_i - M\|$	$f_i\|x_i - M\|$
15.5–20.5	5	5	18	20	100
20.5–25.5	6	11	23	15	90
25.5–30.5	12	23	28	10	120
30.5–35.5	14	37	33	5	70
35.5–40.5	26	63	38	0	0
40.5–45.5	12	75	43	5	60
45.5–50.5	16	91	48	10	160
50.5–55.5	9	100	53	15	135
	$N=\Sigma f_i=100$				$\sum f_i \| x_i - M \| = 735$

The class containing $\frac{N}{2}$th or $\frac{100}{2}$th i.e. 50th observation is 35.5–40.5, which is the median class :

$$\text{Median} = l + \left(\frac{\frac{N}{2} - C}{f}\right) \times h \qquad [l = 35,5, h = 5, C = 37, f = 26]$$

$$= 35.5 + \left(\frac{50 - 37}{26}\right) \times 5 = 35.5 + 2.5 = 38$$

$$\text{M.D(M)} = \frac{\sum f_i |x_i - M|}{N} = \frac{735}{100} = 7.35$$

Example-13 : Find the mean deviation about the mean for the following data :

Height in cm	95 – 105	105 – 115	115 – 125	125 – 135	135 – 145	145 – 155
Number of boys	9	13	26	30	12	10

Solution : We assume a = 130

Height	Number of boys (f_i)	Mid Point (x_i)	$d_i = \frac{x_i - a}{10}$	$f_i d_i$	$\| x_i - \overline{x} \|$	$f \| x_i - \overline{x} \|$
95–105	9	100	–3	–27	25.3	227.7
105–115	13	110	–2	–26	15.3	198.9
115–125	26	120	–1	–26	5.3	137.8
125–135	30	130	0	0	4.7	141
135-145	12	140	1	12	14.7	176.4
145-155	10	150	2	20	24.7	247
	$\Sigma f_i=100$			$f_i d_i = -47$		$\sum f_i \| x_i - \overline{x} \| = 1128.8$

$$\overline{x} = a + \frac{\sum f_i d_i}{N} \times h = 130 + \frac{(-47)}{100} \times 10$$

$$= 130 – 4.7 = 125.3$$

$$\therefore \quad M.D(\overline{x}) = \frac{\sum f_i |x_i - \overline{x}|}{N} = \frac{1128.8}{100} = 11.288$$

Standard Deviation (S.D) and Variance :

The most common and useful measure of dispersion is the standard deviation or variance.

The standard deviation denoted by σ, is defined as the positive square root of the mean of the squares of deviations from the mean $\overline{x}$.

The square of the standard deviation σ^2 is called variance.

Standard deviation for ungrouped data : If $x_1, x_2, \ldots\ldots\ldots, x_n$ be a series of values of a variable and $\overline{x}$ their A.M, then S.D is defined by

$$\sigma = \sqrt{\frac{\sum_{i=1}^{n} (x_i - \overline{x})}{n}}$$

Another formula :

$$\sigma^2 = \frac{1}{n} \sum_{i=1}^{n} (x_i - \overline{x})^2$$

$$= \frac{1}{n} \sum_{i=1}^{n} \left(x_i^2 - 2x_i\overline{x} + \overline{x}^2\right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \frac{2\overline{x}}{n} \sum_{i=1}^{n} x_i + \frac{1}{n} \sum_{i=1}^{n} \overline{x}^2$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i^2 - 2\overline{x}.\,\overline{x} + \frac{\overline{x}^2}{n}.n$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \overline{x}^2$$

$$= \frac{1}{n} \sum_{i=1}^{n} x_i^2 - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2$$

$$\therefore \quad \sigma = \sqrt{\frac{\sum_{i=1}^{n} x_i^2}{n} - \left(\frac{\sum_{i=1}^{n} x_i}{n}\right)^2}$$

Standard deviation of a discrete frequency distribution : Let the given frequency distribution be.

x :	x_1	x_2	x_3	x_n
f :	f_1	f_2	f_3	f_n

Then $\quad \sigma = \sqrt{\frac{1}{N} \sum_{i=1}^{n} f_i (x_i - \overline{x})^2}$

where $N = \sum_{i=1}^{n} f_i$, $\bar{x} = \frac{\sum_{i=1}^{n} f_i x_i}{N}$

Another formula :

$$\sigma^2 = \frac{1}{N}\sum_{i=1}^{n} f_i\left(x_i - \bar{x}\right)^2$$

$$= \frac{1}{N}\sum f_i\left(x_i^2 - 2x_i\bar{x} + \bar{x}^2\right)^2$$

$$= \frac{1}{N}\sum f_i x_i^2 - \frac{1}{N}2\bar{x}\sum x_i + \frac{\bar{x}^2}{N}\sum f_i$$

$$= \frac{1}{N}\sum f_i x_i^2 - 2\bar{x}.\bar{x} + \frac{\bar{x}^2}{N}.N$$

$$= \frac{1}{N}\sum f_i x_i^2 - \bar{x}^2$$

$$= \frac{1}{N}\sum f_i x_i^2 - \left(\frac{\sum f_i x_i}{N}\right)^2$$

$$\therefore \quad \sigma = \sqrt{\frac{1}{N}\sum f_i x_i^2 - \left(\frac{1}{N}\sum f_i x_i\right)^2}$$

Short cut Method :

Let $d_i = x_i - a$, a is assumed mean.

Then $\bar{d} = \bar{x} - a$

$$\therefore \quad d_i - \bar{d} = x_i - \bar{x}$$

$$\sigma = \sqrt{\frac{1}{N}\sum f_i\left(x_i - \bar{x}\right)^2}$$

$$= \sqrt{\frac{1}{N}\sum f_i\left(d_i - \bar{d}\right)^2} = \sqrt{\frac{1}{N}\sum f_i d_i^2 - \left(\frac{1}{N}\sum f_i d_i\right)^2}$$

Standard deviation of a continuous frequency distribution :

If there is a frequency distribution of n classes, each class is defined by its mid-point x_i with frequency f_i, then the standard deviation is given by

$$\sigma = \sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i\left(x_i - \bar{x}\right)^2}$$

where, x_i is the mid value of i^{th} class interval.

Another formula :

$$\sigma = \sqrt{\frac{1}{N}\sum f_i x_i^2 - \left(\frac{1}{N}\sum f_i x_i\right)^2}$$

Short cut method :

Let $y_i = \frac{x_i - a}{h}$

$\Rightarrow \quad x_i = a + hy_i$

$\Rightarrow \quad x_i = a + h\overline{y}$

$\therefore \quad x_i - \overline{x} = h(y_i - \overline{y})$

Variance of the variable x :

$$\sigma_x{}^2 = \frac{1}{N}\sum_{i=1}^{n} f_i (x_i - \overline{x})^2$$

$$= \frac{1}{N}\sum f_i \left\{h(y_i - \overline{y})\right\}^2$$

$$= h^2 \left[\frac{1}{N}\sum f_i \left\{h(y_i - \overline{y})\right\}^2\right]$$

= $h^2 y$ variance of the variable y

$$= h^2 \sigma_y^2$$

$$\therefore \quad \sigma_x = h\sigma_y = h\sqrt{\frac{1}{N}\sum f_i y_i{}^2 - \left(\frac{\sum f_i y_i}{N}\right)^2}$$

where $y_i = \frac{x_i - a}{h}$

Example-1 : The marks obtained by 7 students are :

8, 9, 11, 13, 14, 15, 21

Calculate the mean, variance and standard deviation

Solution : Here $\overline{x} = \frac{8+9+11+13+14+15+21}{7} = \frac{91}{7} = 13$

Now, to find the variance we construct the following table

x_i	$x_i - \overline{x}$	$(x_i - \overline{x})^2$
8	–5	25
9	–4	16
11	–2	4
13	0	0
14	1	1
15	2	4
21	8	64

Here n = 7 $\quad (x_i - \overline{x})^2 = 114$

$\therefore$ Variance of marks $\sigma^2 = \frac{\sum(x_i - \overline{x})^2}{n} = \frac{114}{7} = 16.29$

Standard deviation of marks, $\sigma = \sqrt{16.29} = 4.04$

Example-2 : Find the mean and variance of the first n natural numbers.

Solution : The first n natural numbers are 1, 2, 3, ...,n

$\therefore$ x_i : 1, 2, 3, ... n

$$\text{Mean, } \bar{x} = \frac{\sum x_i}{n} = \frac{1+2+3+......+n}{n} = \frac{\frac{n(n+1)}{2}}{n} = \frac{n+1}{2}$$

$$\text{Variance, } \sigma^2 = \frac{1}{n}\sum x_i^2 - \left(\frac{\sum x_i}{n}\right)^2$$

$$= \frac{1}{n}\left(1^2+2^2+3^2+..........+n^2\right) - \left(\frac{1^2+2^2+3^2+..........+n^2}{n}\right)^2$$

$$= \frac{1}{n}\frac{n(n+1)\,(2n+1)}{6} - \left\{\frac{n(n+1)}{2a}\right\}^2$$

$$= \frac{n(n+1)\,(2n+1)}{6} - \frac{(n+1)^2}{4}$$

$$= \frac{n(n+1)\,(2n+1) - 3(n+1)^2}{12}$$

$$= \frac{(n+1)\,(4n+2-3a-3)}{12}$$

$$= \frac{(n+1)\,(n-1)}{12} = \frac{n^2-1}{12}$$

Example-3 : Find the mean and variance of the first 10 multiple of 3.

Solution : The first 10 multiple of 3 are 3, 6, 9, 12, 15, 18, 21, 24, 27, 30

x_i	$x_i - \bar{x}$	$(x_i - \bar{x})^2$
3	–13.5	182.25
6	10.5	110.25
9	–7.5	56.25
12	–4.5	20.25
15	–1.5	2.25
18	1.5	2.25
21	4.5	20.25
24	7.5	56.25
27	10.5	110.25
30	13.5	182.25
		$\sum(x_i - \bar{x})^2 = 742.5$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{3+6+9+12+15+18+21+24+27+30}{10} = \frac{165}{10} = 16.5$$

$$\text{Variance, } \sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = \frac{742.5}{10} = 74.25$$

Example-4 : The scores of a batsman in 10 matches were as follows :

38, 70, 48, 34, 42, 55, 63, 46, 54, 44

Compute the variance and standard deviation.

Solution : Let us take a = 48 as the assumed mean. The calculation work for finding the variance has been shown in the following table :

Variable x_i	**Deviation** $d_i = x_i - a = x_i - 48$	**Square of deviations** d_i^2
38	–10	100
70	22	484
48	0	0
34	–14	196
42	–6	36
55	7	49
63	15	225
46	–2	4
54	6	36
44	–4	16
Total	$\Sigma d_i = 14$	$\Sigma d_i^2 = 1146$

Here n = number of observations = 10, $\Sigma d_i = 14$ and $\Sigma d_i^2 = 1146$

$$\bar{x} = \frac{\sum x_i}{n} = \frac{3+6+9+12+15+18+21+24+27+30}{10} = \frac{165}{10} = 16.5$$

$\therefore$ Variance of the scores = $\sigma^2 = \frac{\sum d_i^2}{n} - \frac{\sum (d_i)^2}{n} = \frac{1146}{10} - \left(\frac{14}{10}\right)^2 = 114.6 - 1.96 = 112.64$

Also S.D = $\sigma = \sqrt{112.64} = 10.61$ runs

Example-5 : Find the variance for the following data :

Height (in cm)	**No. of students**	**Height (in cm)**	**No. of students**
138	3	158	27
142	8	162	20
146	15	166	9
150	19	170	4
154	35		

Solution : h = Common difference of values of x = 4. Let us take assumed mean as a = 154.

x_i	f_i	$u_i = \frac{x_i - 154}{4}$	$f_i u_i$	$f_i u_i^2$
138	3	–4	–12	48
142	8	–3	–24	72
146	15	–2	–30	60
150	19	–1	–19	19
154	35	0	0	0
158	27	1	27	27
162	20	2	40	80
166	9	3	27	81
170	4	4	16	64
	140		25	451

Here $N = 140, h = 4, \sum f_i u_i = 25, \sum f_i u_i^2 = 451$

$\therefore$ Variance $\sigma^2 = h^2\left[\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2\right]$

$$= (4)^2 \times \left[\frac{451}{140} - \left(\frac{25}{140}\right)^2\right] = 16\left[3.22 - (0.178)^2\right]$$

$$= 16 \times [3.22 - 0.03] = 16 \times 3.19 = 51.04\, cm^2$$

Note : The column $f_i u_i^2 = f_i u_i . u_i$ is obtained by multiplying the corresponding entries of columns of u_i and $f_i u_i$

Example-6 : Find the S.D. of the following distribution :

x_i	4.5	14.5	24.5	34.5	44.5	54.5	64.5
f_i	1	5	12	22	17	9	4

Solution : We prepare the following table after taking assumed mean a = 34.5 and h = 10.

x_i	f_i	$u_i = \frac{x_i - 34.5}{10}$	u_i^2	$f_i u_i$	$f_i u_i^2$
4.5	1	–3	9	–3	0
14.5	5	–2	4	–10	20
24.5	12	–1	1	–12	12
34.5	22	0	0	0	0
44.5	17	1	1	17	17
54.5	9	2	4	18	36
64.5	4	3	9	12	36
Total	70			22	130

Now, S.D. $\sigma^2 = h\sqrt{\frac{1}{N}\sum_{i=1}^{n} f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2} = 10\sqrt{\frac{130}{70} - \left(\frac{22}{70}\right)^2}$

$$= 10\sqrt{\frac{130 \times 70 - 484}{70 \times 70}} = \frac{10}{70}\sqrt{9100 - 484} = \frac{1}{7}\sqrt{8616} = \frac{92.82}{7} = 13.26$$

Example-7 : Find the mean and the standard deviation for the following data :

Age (years)	**No. of teachers**
25 – 30	30
30 – 35	23
35 –40	20
40 – 45	14
45 – 50	10
50– 55	3

Solution : Let us take a = 42.5

Class	x_i	f_i	$u_i = \frac{x_i - 42.5}{5}$	$f_i u_i$	$f_i u_i^2$
25 – 30	27.5	30	–3	–90	270
30 – 35	32.5	23	–2	–46	92

35 – 40	37.5	20	–1	–20	20
40 – 45	42.5	14	0	0	0
45 – 50	47.5	10	1	10	10
50 – 55	52.5	3	2	6	12
	$N=\Sigma f_i=100$			$\Sigma f_iu_i=140$	$\Sigma f_iu_i^2=404$

Here N = 100, a = 42.5, h = 5, Σf_iu_i = 140, and $\Sigma f_iu_i^2$ = 404

$$\therefore \quad \text{Mean } \bar{x}=a+\frac{h}{N}\sum f_iu_i=42.5+\frac{5}{100}(-140)=42.5-7=35.5 \textit{ years}$$

$$S.D(\sigma)=h\sqrt{\frac{1}{N}\sum f_iu_i^2-\left(\frac{1}{N}\sum f_iu_i\right)^2}$$

$$=5\times\sqrt{\frac{404}{100}-\left(\frac{-140}{100}\right)^2}=5\times\sqrt{4.04-(-1.4)^2}$$

$$=5\times\sqrt{4.04-1.96}=5\times\sqrt{2.08}=5\times1.442=7.21 \textit{ years}$$

Example-8 : Find the arithmetic mean, variance and standard deviation for the following distribution :

Variable x_i	5	10	15	20	25
Frquency f_i	5	6	12	16	11

Solution : We construct the following table :

Variable x_i	***Frequency*** f_i	***Step deviations*** $u_i=\frac{x_i-a}{h}=\frac{x_i-15}{5}$	f_iu_i	u_i^2	$f_iu_i^2$
5	5	–2	–10	4	20
10	6	–1	–6	1	6
15	12	0	0	0	0
20	16	1	16	1	16
25	11	2	22	4	44
Total	N = 50		22		86

Here, a = 15, h = 5

We have,

$$A.M.=a+\left(\frac{\sum f_iu_i}{N}\right)\times h=15+\left(\frac{22}{50}\right)\times5$$

= 15 + 2.2 = 17.2

$$\text{Variance}(x)=\sigma^2=\left[\frac{\sum f_iu_i^2}{N}-\left(\frac{\sum f_iu_i}{N}\right)^2\right]\times h^2=\left[\frac{86}{50}-\left(\frac{22}{50}\right)^2\right]\times5^2$$

$$=\frac{86\times50-22\times22}{50\times50}\times25=\frac{4300-484}{100}$$

$$= \frac{3816}{100} = 38.16$$

and $S.D = \sigma = \sqrt{38.16} = 6.18$

Example-9 : Calculate the mean and standard deviation for the following distribution :

Marks	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90
No. of students	3	6	13	15	14	5	4

Solution : Calculate work for finding the A.M. and S.D

Class interval	***Mid-values*** x_i	***Step deviations*** $u_i = \frac{x_i - a}{h} = \frac{x_i - 55}{10}$	***Frequency*** f_i	$f_i u_i$	$f_i u_i^2$
20 – 30	25	3	3	–9	27
30 – 40	35	–2	6	–12	24
40 – 50	45	–1	13	–13	13
50 – 60	55	0	15	0	0
60 – 70	65	1	14	14	14
70 – 80	75	2	5	10	20
80 – 90	85	3	4	12	36
Total			N = 60	2	134

Here assumed mean $a = 55$, $h = 10$

We have,

$$A.M. = a + \left(\frac{\sum f_i u_i}{N}\right) \times h = 55 + \left(\frac{2}{60}\right) \times 10 = 55 + 0.33\, marks$$

$$\text{Variance(x)} = \sigma^2 = \left[\frac{\sum f_i u_i^2}{N} - \left(\frac{\sum f_i u_i}{N}\right)^2\right] \times h^2$$

$$= \left[\frac{134}{60} - \left(\frac{2}{60}\right)^2\right] \times 60 = \left(\frac{8040 - 4}{60 \times 60}\right) \times 100 = \frac{8036}{36}$$

$$= \frac{2009}{9} = 223.222$$

and $S.D = \sigma = \sqrt{223.222} = 14.94\, marks$

Example-10 : Calculate the mean and standard deviation for the following data :

No. of workers	**Wages per week upto Rs.**
12	15
30	30
65	45
107	60
157	75

202	90
222	105
230	120

Solution : Wages per week upto Rs..... ⇒ Wages less than or equal to Rs. .... Therefore, no. of workers given n first column are cumulative frequencies.

Therefore, converting the given e.f. distribution table into an ordinary frequency distribution table, we have

Class	***Mid-value (x_i)***	f_i	$u_i = \frac{x_i - 67.5}{15}$	$f_i u_i$	$f_i u_i^2$
0 – 15	7.5	12	–4	–48	192
15 – 30	22.5	18	–3	–54	152
30 – 45	37.5	35	–2	–70	140
45 – 60	52.5	42	–1	–42	42
60 – 75	67.5	50	0	0	0
75 – 90	82.5	45	1	45	45
90 – 105	97.5	20	2	40	80
105 – 120	112.5	8	3	24	72
Total	230			–105	733

Here N = 230. Let us take a = 67.5, and h = 15

$$Mean\ (\bar{x}) = a + \frac{h}{N}\sum f_i u_i = 67.5 + \frac{15}{230}(-105) = 67.5 - \frac{15 \times 21}{46}$$

$$= 67.5 - 6.85 = Rs.\ 60.65$$

$$S.D.(\sigma) = h\sqrt{\frac{1}{N}\sum f_i u_i^2 - \left(\frac{1}{N}\sum f_i u_i\right)^2} = 15 \times \sqrt{\frac{733}{230} - \left(\frac{-105}{230}\right)^2}$$

$$= 15 \times \sqrt{2.1869 - 0.2084} = 15 \times \sqrt{2.9785} = 15 \times 1.73 = Rs.\ 25.95$$

Example-11 : Given that σ^2 is the variance of the observations $x_1, x_2, \ldots\ldots\ldots\ldots x_n$, prove that the variance of $ax_1, ax_2, \ldots\ldots\ldots\ldots ax_n$, where a is any number different from zero is $a^2\sigma^2$.

Solution : Given σ^2 is the variance of n observations $x_1, x_2, \ldots\ldots\ldots\ldots x_n$.

$$\therefore \quad \sigma^2 = \frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} - \right)^2 \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (1)$$

when each observation is multiplied by a, then the new set of observation is

$ax_1,\ ax_2,\ \ldots\ldots\ldots\ldots\ ax_n$

$$\therefore \quad \text{Variance of the new set of observations} = \frac{\sum_{i=1}^{n}(ax_i)^2}{n} - \left(\frac{\sum_{i=1}^{n} ax_i}{n} - \right)^2$$

[obtained by changing x_1 to ax_1 in (1)

$$= \frac{a^2 \sum_{i=1}^{n} x_i^2}{n} - a^2 \left(\frac{\sum_{i=1}^{n} x_i}{n} - \right)^2 = a^2 \left[\frac{\sum x_i^2}{n} - \left(\frac{\sum x_i}{n} \right)^2 \right] = a^2 \sigma^2 \quad \text{[Using (1)]}$$

Example-12 : The scores of 48 children in an intelligence test are shown in the following frequency table :

Score	**71**	**76**	**79**	**83**	**86**	**89**	**92**	**97**	**101**	**103**	**107**	**110**	**114**
Frequency	**4**	**3**	**4**	**5**	**6**	**5**	**4**	**4**	**3**	**3**	**3**	**2**	**2**

Calculate the variance σ^2 and find out the number of children whose score lies between $\bar{x} - \sigma$ *and* $\bar{x} + \sigma$

Solution : We prepare the following table by taking assumed mean a = 89

Score (x_i)	***Frequency*** f_i	$d_i = x_i - a$	d_i^2	$f_i d_i$	$f_i d_i^2$
71	4	–18	324	–72	1296
76	3	–13	169	–39	507
79	4	–10	100	–40	400
83	5	–6	36	–30	180
86	6	–3	9	–18	54
89	5	0	0	0	0
92	4	3	9	12	36
97	4	8	64	32	256
101	3	12	144	36	432
103	3	14	196	42	588
107	3	18	324	54	972
110	2	21	441	42	882
114	2	25	625	50	1250
Total	48			69	6853

Now,

$$Mean\ (\bar{x}) = a + \frac{\sum f_i d_i}{\sum f_i} = 89 + \frac{69}{48} = 89 + 1.44 = 90.44$$

and Variance $\sigma^2 = \frac{\sum f_i d_i^2}{\sum f_i} - \left(\frac{\sum f_i d_i}{\sum f_i} \right)^2 = \frac{6853}{48} + \left(\frac{69}{48} \right)^2 = 142.77 - 2.07 = 140.70$

$\therefore \quad \sigma = \sqrt{140.70} = 11.86$

$\Rightarrow \quad \bar{x} - \sigma = 90.44 - 11.86 = 78.56$

and $\quad \bar{x} + \sigma = 90.44 + 11.86 = 102.30$

Hence the number of observations which lies between $\bar{x} - \sigma$ *and* $\bar{x} + \sigma$, between 78.58 and 102.30

= 4 + 5 + 6 + 5 + 4 + 4 + 3 = 31

Example-13 : Suppose that the observations x_1, x_2, ... x_n, are changed to $x_1 + a$, x_2 + a, ..., $x_n + a$ (i.e., A common number a is added to each of the n given numbers). Show that the variance remains unchanged.

Solutions : Let $\bar{x}$ and σ^2 be the mean and variance of the observations x_1, x_2, ... x_n. Then we know that

$$\sigma^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 \qquad \text{......................} \quad (1)$$

When a is added to each of the observations, then we know that a will be added to the mean also, i.e., when x_i is changed to $x_i + a$, then $\overline{x}$ is also changed to $\overline{x} + a$. i.e.

New mean = $\overline{x} + a$

Changing x_i to $x_i + a$ and $\overline{x}$ to $\overline{x} + a$ in (1),

$$\text{New variance } = \frac{1}{n}\sum_{i=1}^{n}\left[\left(x_i + a\right) - \left(\overline{x} + a\right)\right]^2 = \frac{1}{n}\sum_{i=1}^{n}\left(x_i - \overline{x}\right)^2 = \sigma^2 \qquad \text{[By (1)}$$

Therefore, variance remains unchanged.

Example-14 : The mean and variance of 7 observations are 8 and 16 respectively. If 5 of the observations are 2, 4, 10, 12, 14, find the remaining two observations.

Solutions : Let x and y be the remaining two observations. Then

Mean = 8

$$\Rightarrow \quad \frac{2+4+10+12+14+x+y}{7} = 8 \Rightarrow 42 + x + y = 56 \Rightarrow x + y = 14 \qquad \text{.......} (1)$$

Variance = 16

$$\Rightarrow \quad \frac{1}{7}\left(2^2 + 4^2 + 10^2 + 12^2 + 14^2 + x^2 + y^2\right) - (\text{Mean})^2 = 16$$

$$\Rightarrow \quad \frac{1}{7}\left(4 + 16 + 100 + 144 + 196 + x^2 + y^2\right) - 64 = 16$$

$$\Rightarrow \quad 460 + x^2 + y^2 = 7 \times 80 \Rightarrow x^2 + y^2 = 100$$

$$\text{Now,} \quad (x+y)^2 + (x-y)^2 = 2\left(x^2 + y^2\right)$$

$$\Rightarrow \quad 196 + (x-y)^2 = 2 \times 100 \Rightarrow (x-y)^2 = 4 \Rightarrow (x-y)^2 = \pm 2$$

If x – y = 2, then x + y = 14 and x – y = 2

$\Rightarrow$ x = 8, y= 6

Hence the remaining two observations are 6 and 8.

Example-15 : From a frequency distribution consisting of 18 observations, the mean and the standard deviation were found to be 7 and 4 respectively. But on comparison with the original data it was found that a figure 12 was miscopied as 21 in calculations. Calculate the correct mean and standard deviation.

Solutions : (i) Calculation of correct arithmetic mean :

We know that

$$\overline{x} = \frac{\sum x}{n} \qquad \therefore \quad n\overline{x} = \sum x$$

Here $\quad n = 8,\ \overline{x} = 7 \qquad \therefore \quad \sum x = 18 \times 7 = 126$

But this is wrong $\sum x$

$\therefore \quad$ Correct $\sum x = 126 - 21 + 12 = 117 \qquad \therefore \quad$ Correct arithmetic mean $= \dfrac{117}{18} = 6.5$

(ii) Calculation of correct standard deviation :

We know that $\sigma = \sqrt{\dfrac{\sum x^2}{n} - \left(\dfrac{\sum x}{n}\right)^2}$

Or $4 = \sqrt{\dfrac{\sum x^2}{n} - (7)^2}$ *or* $16 = \dfrac{\sum x^2}{18} - (7)^2$ *or* $\dfrac{\sum x^2}{18} = 16 + 49 = 65$

Or $\sum x^2 = 65 \times 18 = 1170$

But this is wrong $\sum x^2$

$\therefore$ Correct $\sum x^2 = 1170 - (21)^2 + (21)^2 = 1170 - 441 + 144 = 873$

$\therefore$ Correct standard deviation

$$= \sqrt{\frac{\text{Correct} \sum x^2}{n} - (\text{Correct mean})^2} = \sqrt{\frac{873}{18} - (6.5)^2}$$

$$= \sqrt{48.5 - (6.5)^2} = \sqrt{6.25} = 2.5$$

Example-16 : The A.M. and S.D. of 100 item was recorded as 40 and 5.1 respectively. Later on it was discovered that one observation 40 was wrongly copied down is 50. Find the correct S.D.

Solutions : No. of items = 100, Incorrect mean$(\bar{x})$ = 40, Incorrect S.D. = 5.1, Incorrect item = 50, Correct item=40

Now, $\bar{x} = \dfrac{\sum x}{n} \Rightarrow 40 = \dfrac{\text{Incorrect} \sum x}{100}$ $\therefore$ Incorrect $\sum x = 4000$

$\therefore$ Correct $\sum x = 4000 - 50 + 40 = 3990$ $\therefore$ Correct mean $= \dfrac{3990}{100} = 39.9$

Now, $S.D. = \sqrt{\dfrac{\sum (x - \bar{x})^2}{n}} = \sqrt{\dfrac{\sum x^2}{n} - (\bar{x})^2}$

$\therefore$ $5.1 = \sqrt{\dfrac{\text{Incorrect} \sum x^2}{100} - (40)^2}$ *or* $26.01 = \dfrac{\text{Incorrect} \sum x^2}{100} - 1600$

$\therefore$ Incorrect $\sum x^2 = 162601$ $\therefore$ Correct $\sum x^2 = 162601 - (50)^2 + (40)^2 = 161701$

$\therefore$ Correct $S.D. = \sqrt{\dfrac{161701}{100} - (39.9)^2} = \sqrt{1617.01 - 1592.01} = 5$

Example-17 : The mean and standard deviation of a group of 10 observations were found to be 20 and 3 respectively. Later on it was found that three observations were incorrect, which were recorded as 21, 21 and 18. Find the mean and standard deviation if the incorrect observations were omitted.

Solutions : Mean $\bar{x} = \dfrac{\sum x_i}{n}$, $\bar{x} = 20$

$\therefore$ $\sum x_i = n \times \bar{x} = 100 \times 20 = 2000$

Incorrect value of $\sum x_i = 2000$

Correct value of $\sum x_i = 2000 - 21 - 18 = 1940$

Correct value of mean $= \frac{1940}{97} = 20$

$$\sigma^2 = \text{(Incorrect)} = \frac{\sum x_i^2}{n} - (\bar{x})^2$$

$$\Rightarrow \quad 9 = \frac{\sum x_i^2}{n} - (20)^2$$

$\therefore \quad \sum x_i^2 = 409 \times 100 = 40900$

Incorrect value of $\sum x_i^2 = 40900 - (21)^2 - (21)^2 - (18)^2$

$= 40900 - 441 - 441 - 324 = 39694$

$\therefore \quad$ Corrected $\sigma^2 = \frac{39694}{97} - (20)^2$

$= 409.22 - 400 = 9.22$

Correct value of standard deviation $\sigma = 3.04$

Example-18 : Given that $\bar{x}$ is the mean and σ^2 is the variance of n observations $x_1, x_2, x_3, ..., x_n$. Prove that the mean and variance of the observations $ax_1, ax_2, ax_3, ..., ax_n$ are $a\bar{x}$ and $a^2\sigma^2$ respectively ($a \neq 0$).

Solutions : Mean of $ax_2, ax_3, ..., ax_n = \frac{ax_1 + ax_2 + ... + ax_n}{n}$

$$= \frac{a(x_1 + x_2 + x_3 + ... + x_n)}{n} = a\bar{x}$$

$$\left[\because \ \bar{x} = \frac{x_1 + x_2 + x_3 + ... + x_n}{n} \text{ is given}\right]$$

Variance of $ax_1, ax_3, ..., ax_n = \frac{\sum (ax_i - a\bar{x})^2}{n}$

$$= \frac{(ax_1 - a\bar{x})^2 + (ax_2 - a\bar{x})^2 + ... + (ax_n - a\bar{x})^2}{n}$$

$$= a^2 \left[\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + ... + (x_n - \bar{x})^2}{n}\right]$$

$$= a^2 \frac{\sum (x_i - \bar{x})^2}{n} = a^2\sigma^2$$

EXERCISE : 15

1. Find the mean deviation from the mean for the following data :
(i) 30, 40, 70, 60, 20, 10, 50
(ii) 4, 7, 8, 9, 10, 12, 13, 17
(iii) 4.5, 4.7, 5, 5.25, 5.5, 6.25, 6.5, 7.75, 8.5
2. Find the mean deviation from the mediam for the following data :
(i) 22, 24, 30, 27, 29, 31, 25, 41, 42
(ii) 34, 66, 30, 38, 44, 50, 40, 60, 42, 51
(iii) 3, 9, 21, 12, 5, 3, 18, 4, 7, 10, 19
3. The mean of 6, 8, 5, 7, x and 4 is 7 find the mean deviation about mediam of these observations.
4. Find the mean deviation from the mediam for the following data :

x_i	5	7	9	11	13	15	17
f_i	2	5	6	8	10	12	8

5. Calculate the mean deviation from the median of the following frequency distribution :

Age	15	13	17	16	18	20
No. of students	4	5	4	2	6	3

6. Compute the mean deviation from the median of the following distribution:

Class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
Frequency	5	10	20	5	10

7. Find the mean deviation from mean and the coeffcient of mean deviation of the following distribution:

Class	1 – 5	6 – 10	11 – 15	16 – 20	21 – 25
Frequency	8	5	20	7	10

8. Find the mean deviation from the mean for the following observations :

Class	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
Frequency	2	3	8	14	8	3	2

9. Find the mean deviation form the median of the following distribution:

(i)

Class	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70
Frequency	3	8	12	9	8

(ii)

Class	0 – 6	6 – 12	12 – 18	18 – 24	24 – 30
Frequecny	8	10	12	9	5

10. Find the mean and variance of the following:
(i) 6, 7, 10, 12, 14, 4, 8, 12
(ii) 65, 58, 68, 44, 48, 45, 60, 62, 60, 50
11. Find the mean, variance and standard deviation of the following marks scored by 10 students :

45, 70, 62, 60, 50, 48, 67, 34, 65, 58

12. The mean and variance of 8 observations are 9 and 9.25 respectively. If six of the observations are 6, 7, 10, 12, 12 and 13, find the remaining two observations.
13. The variance of 20 observations is 5. If each observation is multiplied by 2, find the new variance of the resulting observations.
14. The mean and standard deviation of 6 observations are 8 and 4 respectively. If each observation is multiplied by 3, find the new mean and new standard deviation of the resulting observations.
15. Mean and standard deviation of 100 items are 50 and 4 respectively. Find the sum of all the items and also the sum of the squares of the items.
16. While calculating the mean and variance of 10 readings, a student wrongly used the figure 52 for the correct figure 25. He obtained the mean and variance as 45 and 16 respectively. Find the correct mean and variance.
17. The mean and standard deviation of 200 items are found to be 60 and 20 respectively. If at the time of calculation, two items were wrongly taken as 3 and 67 instead of 18 and 17, respectively, find the correct mean and the standard deviation.
18. Find the mean and standard deviation of the following distribution

Marks	10	20	30	40	50	60	70
No. of students	1	5	12	22	17	9	4

19. Find the mean and standard deviation for the following data :

x_i	92	93	97	98	102	104	109
y_i	3	2	3	2	6	3	3

20. Find the variance and standard deviation for the following data:

(i)

x_i	4	8	11	17	20	24	32
y_i	3	5	9	5	4	3	1

(ii)

x_i	3	8	13	18	23
y_i	7	10	15	10	6

21. The following frequency table given the ages of a group of 50 children invited to a birth day party. Find the mean and standard deviation of the distribution :

Age(in *years*)	5 – 7	7 – 9	9 – 11	11 – 13	13 – 15
Frequency	16	13	10	6	5

22. Find the S.D. of the following data.

Class Interval	25 – 35	35 – 45	45 – 55	55 – 65	65 – 75
Frequecny	21	20	16	25	18

23. Find the mean, variance and S.D using short cut - method:

Class interval	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80	80 – 90	90 – 100
Frequency	3	4	7	7	15	9	6	6	3

24. Find the mean and variance of the following distribution :

Classes	0 – 30	30 – 60	60 – 90	90 – 120	120 – 150	150 – 180	180 – 210
Frequencies	2	3	5	10	3	5	2

25. The marks obtained by 200 pupils of a class in their annual examination are given in the following table. Find the mean and standard deviation of these marks.

Marks	1 – 10	11 – 20	21 – 30	31 – 40	41 – 50	51 – 60	61 – 70	71 – 80	81 – 90	91 – 100
No. of Student	2	3	10	19	30	47	54	28	5	2

26. Following are the marks obtained out of 100 marks by two students in 10 tests. Find who is more intelligent and who is more consistent.

A	25	50	45	30	70	42	36	48	35	60
B	10	70	50	20	95	55	42	60	48	80

27. Goals scored by teams A and B in a football season are as follows. Find which team is more consistent in its performance :

Number of goals scored in one match	0	1	2	3	4
No. of Team A	27	9	8	5	4
MatchsTeam B played	17	9	6	5	3

28. From the prices of shares X and Y below, find out which is more stable in value.

X	35	54	52	53	56	58	52	50	51	49
Y	108	107	105	105	106	107	104	103	104	101

29. From the data given below state which group is more variable, A or B?

Marks	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
GroupA	9	17	32	33	40	10	9
Group B	10	20	30	25	43	15	7

ANSWERS

1. (i) 17.1 (ii) 3 (iii) 1.1
2. (i) 4.7 (ii) 8.7 (iii) 5.27
3. 2
4. 2.72
5. 2.708

6. 9
7. 4.76, $\frac{4.76}{13.5}$
8. 10
9. (i) 19.125 (ii) 6.32
10. (i) 9 (ii) 9.25
11. 55.9, 115.89, 10.77
12. 4, 8
13. 20
14. 24, 12
15. 5000, 251600
16. 42.3, 13.81
17. 59.8, 20.11
18. 13.27
19. 100. 5.39
20. (i) 45.8, 6.77 (ii) 37.45, 6.12
21. 8.84 years, 2.62 years
22. 14.18
23. 56,422.33, 20.55
24. 107, 22.76
25. 54.85, 16.37
26. B is more intelligent, A is more consistent
27. Team B is more consistent
28. Y
29. B

● ● ●