MATHEMATICS

CLASS XI

(Under AHSEC Curriculum)

Content Developed by-

Dr. Dipak Sarma
&
Dr. Parag Kumar Deb
Associate Professor
Dept. of Mathematics
Cotton College, Assam

Dr. Anjana Bhattacharyya
Associate Professor
Dept. of Mathematics
B. Borooah. College, Assam

\

Assam Electronics Development Corporation Limited
(AMTRON)
(A Government of Assam Undertaking)

Name of the textbook

MATHEMATICS

Name of the Chapter

Probability

(Chapter 16)

CONTENTS

CHAPTER
16

PROBABILITY

If an experiment repeated under identical condition may result in various possible outcomes then it is called a trial or a randan experiment. For example : (i) tossing a coin, (ii) tossing two coins simultaneousely, (iii) throwing a die etc. are random experiment.

The set of all possible outcomes of a random experiment is called sample space associated with it, denoted by S.

Each element of the sample space is called a sample point.

For example when a coin is tossed once, there are two possible outcomes - head or tail.

It we denote head by H and tail by T, then the sample space of this experiment is

$S = \{H, T\}$

A subset of the sample space associated with a random experiment is called an event, denoted by E.

For example, consider the experiment of tossing a coin two times. The associated sample space is

$S = \{HH, HT, TH, TT\}$

Then a subset $E = \{HT, TH\}$ of S, which corresponds to the occurence of exactly one head, is called an event.

A subset containing a sample point only is called a simple event.

An event E is said to be certain when E = S and impossible when $E = \phi$,

All possible outcomes of a random experiment considered together are said to be exhaustive. Two or more events associated with a random experiment are called mutually exclusive if the happening of one of them ensures the non happening of the others. Events or outcome of a random experiment are said to be equally likely, if there is no reason for any one event to occur in preference to any other event.

If an event has more than one sample point, it is called a compound event. Compound event is the joint occurence of two or more simple events.

Algebra of events

Since events can be represented as sets, the set operations like union, intersection, complement, difference etc can be applied to events.

(i) Complement of an event

The complement of an event E, denoted by $\bar{E}$ or E' or E^c , is the set of all points of the sample space other than the points occuring in E .

Take the example of "tossing two coins."

The sample space is $S = \{HH, HT, TH, TT\}$

It E is the event "atteast one tail appears",

then $E = \{HT, TH\}$

Then E^c or $'not\ E'$ or E' or $\bar{E}$ is

$\{HH\} = \{No\ Tail\ appears\}$

(ii) The event 'A or B'

"The event 'A or B' denoted by A $\cup$ B, is defined by

$A \cup B = \{w | w \in A \text{ or } w \in B\}$

(iii) The event 'A and B'

The event 'A and B', denoted by $A \cap B$ is defined by

$A \cap B = \{w | w \in A \text{ and } w \in B\}$

(iv) The event 'A but not B'

Then event 'A but no B', denoted by A – B is denoted by A – B is defined by

$A - B = \{w | w \in A \text{ and } w \in B\}$

The outcomes which ensure the happening of a particlar event are said to be cases favourable for that event.

Axiomatic apoproach to probability:

A number $P(w_i)$, called the probability of w_i, can be associated with a sample point w_i, such that

(i) $o \le p(w_i) \le 1$

(ii) $\sum P(w_i) = 1$ for all $w_i \in S$

(iii) $P(A) = \sum P(w_i)$ for all $w_i \in A$, where A is an event containing some out comes w_i probabiity of an event :

For a finite sample space with equally likely outcomes,

Probability of an event $P(A) = \frac{n(A)}{n(S)}$

where $n(A) =$ number of outcomes favourable to event **A**

$n(S) =$ total number of possible outcomes.

If A and B are two events, then

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

If A and B are mutually exclusive, then

$P(A \cup B) = P(A) + P(B)$

It A is any event, then

$P(\text{not } A) = P(A') = 1 - P(A)$

Example 1. Describe the sample space for each of the following experiments

(i) A coin is tossed three times

(ii) Two coins are tossed.

(iii) A coin is tossed and a die is thrown.

(iv) A com is tossed thrice and the number of heads are recorded.

(v) A card is drown from a deck of playing cards, and its colour is noted.

(vi) A coin is tossed and there a die is rolled only in case a head is shown in the coin.

Sol^n :

(i) When a coin is tossed thrice, each toss can come up with either a head or a tail. It we denot head by 'H' and tail by T, then the sample space of this experiment is

$\{HHH, HTH, HTT, HHT, THT, TTH, THH, TTT\}$

(ii) When two coins are tossed, either coin come up with either head (H) or tail (T) The possible outcomes may

be -
Heads on both coins = HH
head on the 1st and tail on the 2nd = HT
tail on the 1st and head on the 2nd = TH
taidl on both coins = TT

∴ The sample space is $\{HH, HT, TH, TT\}$

(iii) When a coin is tossed and together with it a dice is thrown, each outcome can be denoted one line missing head (H) or tail (T) appeared in the coin any y is the number appeared in the dice. y can take any number of 1, 2, 3, 4, 5, 6. Hence the sample space contain 2 ×6 =12 elements and is given by

$$\{(H,1),(H,2),(H,3),(H,4),(H,5),(H,6),(T,1),(T,2),(T,3),(T,4),(T,5),(T,6)\}$$

(iv) When a coin is tossed thrice, head (H) may not appear in any one of the toss. two toss or atmost in all three toss.
Hence number of heads appear may be 0, 1, 2, 3,

∴ Sample space is $\{0,1,2,3\}$

(v) When a card is drawn from a dieck of playing cards, the colour of the card may be red or black. If we donote red by R and black by B, then the sample space of the event is $\{B, R\}$

(vi) When a coin is tossed, the two possible out comes are head (H) or tail (T).
When head appeared a dice is also thrown, in which possible outcomes are 1,2,3,4,5,6 . Thus when head (H) appeared, the possible outcomes of the event are H1, H2, H3, H4, H5, H6, when tail (T) appeared, dice not been thrown
Hence the sample space is

$$\{H1, H2, H3, H4, H5, H6, T\}$$

Ex. 2 Two boys and two girls are in room X and 1boy and 3 girls are in room Y. Specify the sample space for the experiment in which a room is selected and then a person.

Sol[n] : Let the 2 boys and 2 girls in room are marked as B_1, B_2 and G_1, G_2 respectively. Also 1 boy and 3 girls in room Y are marks as B_3 and G_3 , G_4 , G_5 respectively.
Hence the sample space is

$$\{XB_1, XB_2, XG_1, XG_2, YB_3, YG_3, YG_4, YG_5\}$$

Ex. 3 : A box contains 1 red and 3 identicald white balls. Two balls are drawn at random in succession without replacement. Write the sample space for this experiment.
Solution. Let us donote red bal by R and each while ball by W. When two balls are drawn at Random in succession without replacements, the possible outcomes may be.
1st one red and 2nd one while = RW
1st one white and 2nd one red = WR
both are white = WW

∴ The sAmple space is $\{RW, WR, WW\}$

Ex. 4 A die is thrown repeatedly until a six comes up. What is the sample space for this experiment.
Sol[n] : When a die is thrown repeatedly, a six may come up in 1st throw, 2nd throw, 3rd throw ... and so on.
When 6 come up in the 1st throw, possible outcome is 6.
When 6 come up in the 2nd throw, possible outcomes are (1, 6),(2, 6), (3, 6), (4, 6), (5, 6),
When 6 come up in the 3rd throw, possible outcomes are (1, 1,6), (1, 2, 6), (1, 3, 6) ....
(2, 1, 6) .... (3, 1, 6) ..... (5, 1, 6) .... etc. and so on.

Hence the sample space is
{ 6, (1,6), (2, 6), ... (5, 6), (1,1,6) .... (1, 5, 6) (2, 1, 6) ...... , (5, 1, 6) ....}

Ex. 5 A coin is tossed. If it shows head, we draw a ball from a bag consisting to 2 blue and 3 white balls, if it shows tail, we toss a coin again. Describe the sample space.

Sol[n] : When a coin is tossed, the possible outcomes are head (H) and tail (T) .

Let 2 blue and 3 white balls in the bag are denoted by B_1, B_2 and W_1, W_2, W_3 respectively.

When H appears in the coin, a ball is drawn from the bag. The possible outcomes are

$HB_1, HB_2, HW_1, HW_2, HW_3$

When T appears in the coin, a coin is tossed again. Then the possible ot comes are TH and HT.
Hence the sample space is

$\left\{HB_1, HB_2, HW_1, HW_2, HW_3, TH, TT\right\}$

Ex.6. Give an example of an impossible event.

Sol[n] : E = {Getting a number greater then 6}
when a die as thrown in an example of an impossible event. Beacasuse when a die is thrown the possible outcomes are 1, 2, 3, 4, 5, 6

Ex.7. A die is rolled. Let E be the event "die shows 4" and F be the event "die shows even number." Are E and F mutually exclusive?

Sol[n]: When a die is rolled the sample space is $S = \left\{1, 2, 3, 4, 5, 6\right\}$

The event E which is "die shows 4" can be represented as

$E = \left\{4\right\}$

The event F which is "die shows even number " can be represented as

$F = \left\{2, 4, 6\right\}$

Then $E \cap F = \left\{4\right\}$

$\Rightarrow$ Events E and F are not mutually exclusive

Ex.8. Three coins are tossed once. Let A donote the event "three heads show", B denote the event "two heads and are tail show", C donote the event "three tails show and D denote the event "a head shows on the first coin". Which events are
(i) mutually exculsive?
(ii) simple?
(iii) compound?

Sol[n]: Three coins are tossed once. Then the sample space is

$S = \left\{HHH, HTH, HHT, HTT, THH, THT, TTH, TTT\right\}$

Given,

$A = \left\{HHH\right\}$

$B = \left\{HTH, HHT, THH\right\}$

$C = \left\{TTT\right\}$

$D = \left\{HHH, HTH, HTT, HHT\right\}$

Then
(i) Events A and B , A and C, B and C, C and D are mutually exclusive.
(ii) A and C are simple

(iii) B and D are compound

Ex.9. Two dice are thrown. If E is the event "both dice come up with same number" and F is the event "multiple of the numbers on the two dice is odd'', then describe.

(i) E (ii) F (iii) E or F (iv) E and F (v) E but not F

Sol^n: When two dice are thrown,

(i) Event E is represented as

$E=\{(1,1),(2,2),(3,3)(4,4)(5,5)(6,6)\}$

(ii) The event F is represented as

$F=\{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$

(iii) E or F is

$E\cup F=\{(1,1),(2,2),(3,3),(4,4),(5,5),(6,6),(1,3),(1,5),(3,1),(3,5),(5,1),(5,3)\}$

(iv) E and F on $E\cap F=\{(1,1),(3,3),(5,5)\}$

(v) E but not F or $E-F=\{(2,2),(4,4),(6,6)\}$

Ex.10. A coin is tossed thrice. If E donotes the 'number of heads is odd and F donotes the. 'number of tails is odd', then find the cases favourable to the event $E\cap F$

Sol^n: When a coined is tossed thrice, then the sample space is

$S=\{HHH,HTH,HHT,HTT,THH,THT,TTH,TTT\}$

The event E is represented by

$E=\{HHH,HTT,THT,TTH\}$

Then even F is represented by

$F=\{HTH,HHT,THH,TTT\}$

$\therefore$ $E\cap F=\phi$

i.e there are no cases favourable to $E\cap F$

Ex.11. From a group of 2 men and 3 women, two persons are selected. Decribe the sample space of the experiment. If E is the event in which one man and one woman are selected, then which are the cases favourable to E.

Sol^n: Let 2 men are denoted by M_1 and M_2 and 3 women are donoted by W_1, W_2 and W_3. The sample space of the experiment of selecting two persons is

$S=\{M_1M_2,M_1W_1,M_1W_2,M_1W_3,M_2W_1,M_2W_2,M_2W_3,W_1W_2,W_1W_3,W_2W_3\}$

It E is the event in which one man and one woman are selected, then the cases favourable to E are .

$E=\{M_1W_1,M_1W_2,M_1W_3,M_2W_1,M_2W_2,M_2W_3\}$

Ex.12. Let a sample space be

$S=\{w_1,w_2,w_3,w_4,w_5\}$. which of the following assignments of probability to each outcome are valid.

Outcome :	w_1	w_2	w_3	w_4	w_5
(i)	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$

(ii)	$\frac{1}{2}$	$\frac{3}{5}$	$-\frac{1}{5}$	$\frac{1}{5}$	$\frac{1}{5}$
(iii)	0	0	1	0	0
(iv)	0.1	0.2	0.3	0.4	0.5

Sol[n]: (i) Here each $P(\omega_i)$ is between 0 and 1, and $\sum P(w_i)=1$

Hence the given assignment is valid

(ii) Here $P(w_3)$ is negative. Hence the given assignment of probabilities is not valid.

(iii) Here each $P(\omega_i)$ is between 0 and 1 and $\sum P(w_i)=1$

Hence thegiven assignment is valid.

(iv) Here each $P(\omega_i)$ is between 0 and 1 but $\sum P(\omega_i)\neq 1$

Hence the given assignment is not valid.

Ex.13. A coin is tossed twice what is the probability that atleast one tail occures

Sol[n]: When a coin is tossed twice, the sample space is

$S=\{HH, HT, TH, TT\}$

The cases favourable to the event "atleast one tail occurs" are $\{HT, TH, TT\}$

Let this event be A

Then $P(A)=\dfrac{cases\ \ favourable\ of\ A}{total\, no.\ of\ \ possible\ outcomes}$

$=\dfrac{3}{4}$

Ex.14. If $\frac{2}{11}$ is the probability of an event A, then what is the probability of the event 'not A'

Sol[n]: Here total number of possible out comes of an experiment is 11.

cases favourable to an event A is 2

So, cases favourable to event "not A" or A/ is $11-2=9$

$\therefore \quad P(A')=\dfrac{9}{11}$

Altier we have $P(A')=1-P(A)$

$=1-\dfrac{2}{11}$

$=\dfrac{9}{11}$

Ex.15. There are four men and six women on the city council. If one council member is selected then what is the probability that it is a women.

Sol[n]: With four men and six women, there are 10 members on the city council. It one member is selected, total number of possible out comes will be 10.

Let A be the event "the selected member is a women."

Then favourable cases for A will be 6

$\therefore \quad P(A)=\frac{6}{10}=\frac{3}{5}$

Ex.16. A card is selected from a pack of 52 cards.

(a) How many points are there in the sample space?

(b) Calculate the probability that the card is an ace of spades.

(c) Calculate the probability that the card is

(i) an ace (ii) black card

Sol^n: A card is selected from a pack of 52 cards. Then

(a) There are 52 points in the sample space

(b) There is only one ace of spades in the pack

It A is the event "the selected card is an ace of spade" then

$$P(A)=\frac{1}{52}$$

(c) (i) There are 4 aces in the pack. It B is the event "the selected card is an ace" then

$$P(C)=\frac{26}{52}=\frac{1}{2}$$

Ex. 17. A fair coin with 1 marked on one face and 6 on the other and a fair die are both tossed. Find the probability that the sum of numbers that turn up is (i) 3, (ii) 12.

Sol^n: When the fair coin and the fair die are tossed together, the sample space is given by

$$S=\{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),(6,1),(6,2),(6,3),(6,4),(6,5),(6,6)\}$$

Which contain 12 points

(i) The event "the sum of numbers that turn up is 3" be denoted by A then total number of points in set A is 1, as the favourable case for A is (1,2) only

$\therefore \quad P(A)=\frac{1}{12}$

(ii) The event ''the sum of numbers that turn up is 12'' be denoted by B.

Then total number of points in set B is 1, as the favourable case for B is(6, 6) only.

$\therefore \quad P(B)=\frac{1}{12}$

Ex.18. A fair coin is tossed four times, and a person win Rs 1 for each head and lose Rs. 1.50 for each tail that turns up. From the sample space calculate how many different amounts of money you can have after four tosses. and probability of having each of these amounts?

Sol^n: When a fair coin is tossed 4 times, the sample space is given by

$$S=\{HHHH,HTHH,HTTH,HTTT,HHTT,HHHT,HTHT,HHTH,THHH,TTHH,TTTH,TTTT,THTT,THHT,TTHT,THTH\}$$

which contain 16 points

A person win Rs 1 for each head and lose Rs. 1.50 for each tail that turns up in the toss. For the event "occurence of 4 heads", denoted by A, the person win Rs. 4.00

For the event "ocurence of 3 heads and 1 tail", denoted by B, the person win Rs 1.50. For the event "occurence of 2 heads and 2 tails", denoted by C, the person lose Rs 1.00 For the event "occurence of 4 tails'', denoted by E, the person lose Rs. 6.00

From the sample space we observe that favourable cases for the events A, B, C, D, E are 1, 4, 6, 4, 1 respectively.

$\therefore \quad P(A)=\frac{1}{16}, \quad P(B)=\frac{4}{16}=\frac{1}{4} \quad P(C)=\frac{6}{16}=\frac{3}{8}$

$P(D)=\frac{4}{16}=\frac{1}{4} \quad P(E)=\frac{1}{16}$

Ex. 19. In a single throw of two dice, determine the probability of not getting the same number on the two dice.

Sol^n: In a a single throw of two dice, the sample space is

$$S=\{(1,1),(1,2),.....(1,6),(2,1),(2,2).,...(2,6),$$
$$(3,1),(3,2),.....(3,6),(4,1),(4,2)......(4,6),$$
$$(5,1),(5,2),.....(5,6),(6,1),(6,2)......(6,6)\}$$

Which contain 36 points

It A is the event “not getting the same number on the two dice” then A contain 30 points

$\therefore \quad P(A)=\frac{30}{36}=\frac{5}{6}$

Ex.20. Given $P(A)=3/5$, $P(B)=1/5$ Find P (A or B), if A and B are mutually exclusive events.

Sol^n: When A and B are mutually exclusive, i.e. $A\cap B=\phi$ then

$P(A \text{ or } B)=P(A\cup B)=P(A)+P(B)$

$=\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$

Ex.21. The probability that a student will pass the final examination in both English and Hindi is 0.5 and the probability of passing neither is 0.1. It the probability of passing the English examination is .75, what is the probability of passing the Hindi examination?

Sol^n: Let E be the event of “passing English examination” and H be the event of “passing Hindi examination”

Given P (E∩ H) = 0.5

P(E) = 0.75

$P\left((E\cup H)'\right)=0.1$

$\therefore \quad P(E\cup H)=1-P\left((E\cup H)'\right)$

$=1-0.1=0.9$

We know

$P(E\cup H)=P(E)+P(H)-P(E\cap H)$

$\Rightarrow 0.9=0.75+P(H)-0.5$

$\Rightarrow P(H)=0.9-0.25$

$=0.65$

Ex.22. 4 cards are drawn from a well shuffled deck of 52 cards. What is the probability of obtaining 3 diamonds and 1 spade.

Sol^n: When 4 cards are drawn from a deck of 52 cards, total number of possible outcomes $=52_{C_4}$ Which are equally likely

As there are 13 diamond and 13 spade cards, so 4 cards containing 3 diamonds and 1 spade can be obtain in $13_{C_3}.13_{C_1}$ different ways

Hence the required probability

$$=\frac{13_{C_3}.13_{C_1}}{52_{C_4}}$$

Ex.23. The probability of happening of one of two exhaustive and mutually exclusive events is $\frac{3}{4}$ time the probability of the happing of the other. Find the probabilities of the two events.

Sol^n: Let the two events be A and B.

Such that $P(A)=\frac{3}{4}P(B)$

Since the events are exhaustive as well as mutually exclusive, so

$$P(A)+P(B)=1$$

$$\Rightarrow \frac{3}{4}P(B)+P(B)=1$$

$$\Rightarrow P(B)=\frac{4}{7}$$

Ans so $P(A)=1-P(B)$

$$=1-\frac{4}{7}$$

$$=\frac{3}{7}$$

Ex.24. A and B are two events such that $P(A)=0.54$ $P(B)=0.69$ and $P(A\cap B)=0.35$

Find (i) $P(A\cup B)$ (ii) $P(A'\cap B')$ (iii) $P(A\cap B')$ (iv) $P(B\cap A')$

Sol^n: Given $P(A)=0.54$ $P(B)=0.69$ $P(A\cap B)=0.35$

(i) we known

$$P(A\cup B)=P(A)+P(B)-P(A\cap B)$$

$$=0.54+0.69-0.35$$

$$=0.88$$

(ii) $P(A'\cap B')=1-P(A\cup B)$ as $A'\cap B'=(A\cup B)'$

$$=1-0.88$$

$$=0.12$$

(iii) We have $P(A\cap B') = P(A-B)$

Also $P(A) = P(A-B) + P(A\cap B)$

$\Rightarrow P(A-B) = P(A) - P(A\cap B)$

$\Rightarrow P(A\cap B') = 0.54 - 0.35$

$= 0.19$

(iv) We have $P(B\cap A') = P(B-A)$

Also $P(B) = P(B-A) + P(A\cap B)$

$\Rightarrow P(B-A) = P(B) - P(A\cap B)$

$\Rightarrow P(B\cap A') = 0.69 - 0.35$

$= 0.34$

Ex.25. A bag contains 6 red, 4 white and 8 blue balls. It three balls are drawn at random, find the probability that one is red, one is whilte and one is blue.

Sol^n: With 6 red, 4 white and 8 blue balls, the bag contain 18 balls.

It three balls are drawn at random, the total number of possible outcomes $= 18_{C_3}$

$= \frac{\lfloor 18}{\lfloor 15 . \lfloor 3} = \frac{18.17.16}{3.2} = 3\times 17\times 16$

Again, the number of ways of drawing 1 red, 1 white and 1 blue ball $= 6_{C_1}\times 4_{C_1}\times 8_{C_1}$

$= 6\times 4\times 8$

$\therefore$ Required probability $= \frac{6\times 4\times 8}{3\times 17\times 16} = \frac{4}{17}$

Ex.26. In a certain lottery 10,000 tickets are sold and ten equal prizes are awarded. What is the probability of not getting a prize it you buy (a) one ticket (b) two tickets (c) ten tickets

Sol^n: There are 10,000 tickets and 10 equal prizes. 10 prizes will be awarded from 10 tickets.
So remaining 9990 tickets will not award any prize.

(i) When 1 ticket is bought, the probability of not getting any prizes $= \frac{9990}{10,000} = \frac{999}{1000}$

(ii) When 2 tickets are bought,

the probability of not getting any prizes $= \frac{9990_{C_2}}{10,000_{C_2}}$

(iii) When 10 tickets are bought,

the probability of not getting any prizes $= \frac{9990_{C_{10}}}{10,000_{C_{10}}}$

Ex.27. The probability that a person will travel by plane is $\frac{1}{4}$ and that he will trave by train is $\frac{3}{5}$. What is the probability that he will travel by plane or train?

Sol^n: Let A be the event “the person travel by plane” and B be the event “the person travel by train.”
Since the person canot travel by plane and train simultaneousely, so A and B are mutually exclusive.

$\therefore \quad A \cap B = \phi \Rightarrow P(A \cap B) = \phi$

Given $P(A) = \frac{1}{4}, P(B) = \frac{3}{5}$

$\therefore \quad P(A \text{ or } B) = P(A \cup B) = P(A) + P(B)$

$= \frac{1}{4} + \frac{3}{5} = \frac{17}{20}$

Ex.28. Out of 100 students, two sections of 40 and 60 are formed. If you and your freiend are among the 100 students, what is the probability that
(i) you both enter the same section
(ii) you both enter the different sections.

Sol^n: The total number of ways of forming two sections of 40 and 60. Students out of 100 students $= 100_{C_{40}}$
(because if we form a section of 40 students out of 100 students, the other rection of 60 students is automatically formed).
(i) It you and your friend enter in the section of 40 students, then we are to choose 38 more students from remaining 98 students.
This can be done in $98_{C_{38}}$ different ways. Similarly, if you and your friend enter in the section of 60 students, then the number of different ways of forming this section $= 98_{C_{58}}$
Hence number of ways of forming two sections of 40 and 60 students so that both you and your friend enter in the same section

$= 98_{C_{38}} + 98_{C_{58}}$

$\therefore$ Required probability $= \frac{98_{C_{38}} + 98_{C_{58}}}{100_{C_{40}}}$

$$= \frac{\frac{\underline{|98}}{\underline{|38}\ \underline{|60}} + \frac{\underline{|98}}{\underline{|58}\ \underline{|40}}}{\frac{\underline{|100}}{\underline{|40}\ \underline{|60}}}$$

$$= \frac{\underline{|98}}{\underline{|38}\ \underline{|60}} \times \frac{\underline{|40}\ \underline{|60}}{\underline{|100}} + \frac{\underline{|98}}{\underline{|58}\ 4} \times \frac{\underline{|40}\ \underline{|60}}{\underline{|100}}$$

$$= \frac{40.39}{100.99} + \frac{60.59}{100.99}$$

$$= \frac{85}{165} = \frac{17}{33}$$

(ii) Probability that you and your friend enter different sections
= 1- probability that both enter in the same section

$$= 1 - \frac{17}{33} = \frac{16}{33}$$

Ex.29. A and B appear in an interview for two vacant posts. Their chances for being selected are $\frac{1}{6}$ and $\frac{1}{4}$. Find the probability that

(i) both are selected (ii) Only one is selected

(iii) atleast one is selected.

Sol[n]: Given

Probability of selection of $A, P(A) = \frac{1}{6}$

Probability of selection of B, $P(B) = \frac{1}{4}$

$\therefore$ $P(\bar{A}) =$ Probability of not selecting **A**

$$= 1 - P(A) = 1 - \frac{1}{6} = \frac{5}{6}$$

$P(\bar{B}) =$ Probability of not selecting B

$$= 1 - P(B) = 1 - \frac{1}{4} = \frac{3}{4}$$

Hence

(i) Probability that both are selected is

$$P(A \cap B) = P(A)P(B)$$

$$= 1/6 \cdot 1/4 = \frac{1}{24}$$

(ii) When only one of A and B are selected,

$$P(A \cap \bar{B}) + P(\bar{A} \cap B) = P(A)P(\bar{B}) + P(\bar{A})P(B)$$

$$= \frac{1}{6} \times \frac{3}{4} + \frac{5}{6} \times \frac{1}{4}$$

$$= \frac{8}{24} = \frac{1}{3}$$

(iii) When atleast one of A and B is selected

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{1}{6} + \frac{1}{4} - \frac{1}{24}$$

$$= \frac{9}{24} = \frac{3}{8}$$

EXERCISE

1. Describe the sample space for the following experiments
   (i) One coin is tossed twice
   (ii) A card is drawn from a deck of playing cards, and it's suit is noted.
   (iii) Two children are selected from a group of 3 boys and 2 girls
2. 3 bulbs are selected at random from a lot. Each bulb is tested and classified as defective (D) or non defective (N).What is the sample space of this experiment?
3. A coin is tossed. It the outcome is a head, a die is thrown. It the die shows up an even number, the die is thrown again. Write the sample space of the experiment.
4. In team A, there are 3 boys and 2 girls. First team is chosen and then a participant. Descirbe the sample space.
5. Two dice are rolled. Let A, B, C be the events of getting a sum 2, sum 3, and a sum 4 respectively.
   (i) Is event A simple ? (ii) Is event B simple ?
   (iii) Is event C compound? (iv) Are events A and B mutually exclusive?
6. A die is thrown twice. Describe the sample space of this experiment. Let E_1 be the event "both numbers are even". E_2 be the event "both numbers are odd" and E_3 be the event "sum is less than 6."
   Describe $E_1, E_2, E_3, E_1 \cup E_{2,} E_1 \cap E_2, E_1 \cup E_3 \quad E_1 \cap E_3 \ E_1^{/}, E_2^{/}$
7. A coin is tossed thrice. If E denotes "the number of heads is odd" and F denotes "the number of tails is odd". then find the cases favourable to the event $E \cap F$.
8. Three coins are tossed. Describe three events which are mutually exclusive and exhaustive.
9. Two dice are thrown together. What is the probability that the sum of the numbers on the two faces is neither divisible by 3 nor by 4.
10. A and B are two independent events. If $P(AB) = \frac{1}{6}$ and $P(\bar{A}\bar{B}) = \frac{1}{3}$. Find P(A) and P(B)
11. 5 coins are picked up at random from a money bag containing two 50p coins , four25p coins and six 10p coins. Find the probability that the total amount drawn is atleast R. 1.50.
12. Three numbers are chosen at random from first 100 natural numbers. What is the probability that these three numbers are consecutive
13. If two letters are chosen at random from the English alphabet, find the probability that both are vowels.
14. Find the probability of drawing all four cards of the same numbers, when four cards are drawn from a pack of 52 playing cards.
15. Among 15 players, 8 are batsman and 7 are bowlers, find the probability that a team is chosen of 6 batsman and 5 bowlers.
16. A single letter is selected at random from the word 'PROBABILITY'. Find the probability that it is a vowel.
17. The probability of A to fail in an examination is 0.2 and that for B is 0.3 Find the probability that either A or B fail in the examination.
18. Three idential dice are rolled. Find the probability that the same number will appear on each of them.
19. An urn contains 5 blue and an unknown number x of red balls. Two balls are drawn at random. If the probability of both of them being blue is $\frac{5}{14}$, find x.
20. A letter is chosen at random from the word "ASSASSINATION". What is the probability that it is a
   (i) vowel? (ii) Consonant?

ANSWERS

1. **(i)** $\{HH, HT, TH, TT\}$

   (ii) $\{Spade,\ Heart,\ Diamond,\ C\text{lub}\}$

   (iii) $\{B_1B_2, B_1B_3, B_1G_1, B_1G_2, B_2G_1, B_2G_2, B_3G_1, B_3G_2, G_1G_2\}$

2. $\{DDD, DDN, DND, DNN, NDD, NDN, NND, NNN\}$

3. $\{T, H1, H3, H5, H21, H22, H23, H24, H25, H26$
   $H41, H42, H43, H44, H45, H46, H61, H62, H63$
   $H64, H65, H66\}$

4. $\{AB_1, AB_2, AB_3, AG_1, AG_2, AB_4, AG_3AG_4,$
   $BB_1, BB_2, BB_3, BG_1, BG_2, BB_4, BG_3, BG_4\}$

5. (i) Yes, (ii) No (iii) Yes (iv) Yes

6. $S = \{(x, y) : x = 1, 2, 3, 4, 5, 6, \& y = 1, 2, 3, 4, 5, 6,\}$

   $E_1 = \{(2,2),(2,4),(2,6),(4,2),(4,4,),(4,6),(6,2),(6,4),(6,6)\}$

   $E_2 = \{(1,1),(1,3),(1,5),(3,1),(3,3),(3,5),(5,1),(5,3),(5,5)\}$

   $E_3 = \{(1,1),(1,2,),(1,3),(1,4),(2,1),(2,2),(2,3),(3,1),(3,2),(4,1)\}$

   $E_1 \cup E_2 = \{(x, y) | (x, y) \in E_1 or (x, y) \in E_2\}$

   $E_1 \cap E_2 = \phi$

   $E_1 \cup E_3 = \{(x, y) | (x, y) \in E_1 or (x, y) \in E_3\}$

   $E_1 \cap E_3 = \{(2,2)\}$

   $E_1^{/} = \{those\ cases\ except\ when\ both\ are\ even\}$

   $E_2^{/} = \{those\ cases\ except\ when\ both\ are\ odd\}$

7. O as $E \cap F = \phi$

8. $A = \{HHH, HHT, HTH, THH\}$

   $B = \{HTT, THT, TTH\}, C = \{TTT\}$

9. $\frac{4}{9}$

10. $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}$

11. $\frac{7}{132}$

12. $\frac{98}{100_{C_3}}$

13. $\frac{2}{65}$

14. $\frac{1}{20825}$

15. $\frac{8_{C_6} + 7_{C_5}}{15_{C_{11}}}$

16. $\frac{3}{11}$

17. 0.44

18. $\frac{1}{36}$

19. 3

20. $\frac{6}{13}, \frac{7}{13}$

● ● ●