MATHEMATICS
CLASS XI
(Under AHSEC Curriculum)

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MATHEMATICS

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Sequences & Series (Chapter 9)

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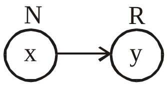
SEQUENCE AND SERIES

A sequence is a function whose domain is the set of N of all natural no. or some subset of it and the range is any set X.

Real sequence : A real sequence 

is a function whose domain in the set N of all natural nos. or some subset of it and range is subset of the real no. R i.e. A real sequence is f : N $\rightarrow$ R

A sequence is denoted by $\{a_n\}_{n=1}^{\propto}$ or $< a_n >_{n=1}^{\propto}$

* f : N $\rightarrow$ R such that

$$fn = \frac{1}{n}, n \in N$$

Then, the sequence is $\left\{1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, ...\right\}$

Series : Let $\{a_n\}_{n=1}^{\propto}$ be a sequence, then $\sum_{n=1}^{\propto} a_n = a_1 + a_2 + a_3 + ...a_n + ...$ to α is called a series. If the no. of terms of the series are infinite that it is called infinite series and the no. of terms of the series are finite are called finite series.

* Write the first five terms of each of the following sequences :

(i) $a_n = n(n+2)$ (ii) $a_n = \frac{n}{n+1}$ (iii) $a_n = (-1)^{n-1}5^{n+1}$ (iv) $a_n = 1 + (-1)^n$

Solution : (i) a_2 = 1(1 + 2) = 3
a_2 = 2(2 + 2) = 8
a_3 = 3(3 + 2) = 15
a_4 = 4(4 + 2) = 24
a_5 = 5(5 + 2) = 35

(ii) $a_1 = \frac{1}{1+1} = \frac{1}{2}$

$$a_2 = \frac{2}{2+1} = \frac{2}{3}$$

$$a_3 = \frac{3}{3+1} = \frac{3}{4}$$

$$a_4 = \frac{4}{4+1} = \frac{4}{5}$$

$$a_5 = \frac{5}{5+1} = \frac{5}{6}$$

(iii) $a_1 = (-1)^{1-1} 5^{1+1} = 5^2 = 25$

$a_2 = (-1)^{2-1} 5^{2+1} = -5^3$

$a_3 = 5^4$

$a_4 = -5^5$

$a_5 = 5^6$

(iv) $a_1 = 1 + (-1)^1 = 0$

$a_2 = 1 + 1 = 2$

$a_3 = 1 - 1 = 0$

$a_4 = 1 + 1 = 2$

$a_5 = 1 - 1 = 0$

Arithmetic Progression (A.P.) : A sequence a_1 , a_2 , a_3 , ..., a_n .... is called arithmetic sequence or arithmetic progression if $a_2 - a_1 = a_3 - a_2 = a_4 - a_3 = \ldots\ldots\ldots\ a_n - a_{n-1} \ldots\ldots\ldots = d$. Here a_1 is called first term and d is called common difference of A.P. nth term of the sequence is $a_n = a_1 + (n - 1)d$.

Example : The first term of an A.P. is $\frac{1}{2}$ and d = 2, find the 15th term.

Solution : Here, $a = \frac{1}{2}$, d = 2

$T_{15} = a + (n - 1)\, d$

$= \frac{1}{2} + (15 - 1) \times 2$

$= \frac{1}{2} + 14 \times 2$

$= \frac{1}{2} + 28 = \frac{5}{2}$

Sum of first 'n' term of A.P.

Let A.P. be

a, a + d, a + 2d, ...

Let, $S_n = a + (a + d) + (a + 2d) + \ldots\ldots\ldots\ldots + \left(a + \overline{n-1}\, d\right)$ .... (1)

Again $S_n = \left(a + \overline{n-1}\, d\right) + \left(a + \overline{n-2}\, d\right) + \ldots\ldots\ldots\ldots + a$ .... (2)

Adding we have,

$$2S_n = \left(2a + \overline{n-1}\, d\right) + \left(2a + \overline{n-2}\, d\right) + \left(2a + \overline{n-2}\, d\right) + \ldots\ldots\ldots\ldots + \left(2a + \overline{n-1}\, d\right)$$

$$= n\left\{2a + \overline{n-1}\, d\right\}$$

$$\therefore \quad S_n = \frac{n}{2}\left\{2a + (n-1)\, d\right\}$$

Example : If $S_n = an^2 + bn$, prove that the series is in A.P.

Solution : $S_n = an^2 + bn$
$S_1 = a + b$
$S_2 = 4a + 2b$
$S_3 = 9a + 3b$
Now, S_1 = first term = a + b
S_2 = Sum of first and 2nd terms
$\Rightarrow$ 4a + 2b = a + b + 2nd term
$\Rightarrow$ 2nd term = 3a + b
S_3 = Sum of first, 2nd and 3rd terms
$\Rightarrow$ 9a + 3b = 4a + 2b + 3rd term
$\Rightarrow$ 3rd term = 5a + b

The sequence is
a + b, 3a + b, 5a + b, ...
3a + b – a – b = 2a
5a + b – 3a – b = 2a

Since, difference of sequence is const
$\therefore$ Series in A.P.

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{n}{2}\{a + a(\overline{n-1})d\}$$

$$= \frac{n}{2}\{\text{First term} + \text{last term}\}$$

Arithmetic Mean :

If three nos. a, b, c are in A.P., then b is called arithmetic mean between a and c. Arithmetic mean is denoted by A.M.

Since, a, b, c are in A.P., therefore
b – a = c – b
$\Rightarrow$ 2b = a + c

$$\Rightarrow b = \frac{a+c}{2}$$

Example : Find the arithmetic mean between 4 & 36.

Solution : A.M. $= \frac{4+36}{2} = 20$

Example : Insert five arithmetic means between 2 and 128.

Solution : Here first term a = 2
128 is the 7th term
Let, d be the c.d
Now, $T_7 = a + (7 - 1)d$
$\Rightarrow$ 128 = 2 + 6d
$\Rightarrow$ 6d = 126
$\Rightarrow$ d = 21
$\therefore$ $T_2 = a + d = 2 + 21 = 23$
$T_3 = a + 2d = 2 + 2 \times 21 = 2 + 42 = 44$
$T_4 = a + 3d = 2 + 3 \times 21 = 2 + 63 = 65$
$T_5 = a + 4d = 2 + 4 \times 21 = 2 + 84 = 86$

$T_6 = a + 5d = 2 + 5 \times 21 = 2 + 105 = 107$

$\therefore$ A.M.s are 23, 44, 65, 86, 107

Example : Find the sum of odd integers from 1 to 2001

Solution : Here, first term, a = 1

d = 2

n = 1000 + 1 = 1001

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$= \frac{1001}{2}\{2 \times 1 + (1001-1) \times 2\}$$

$$= \frac{1001}{2}\{2 + 2000\}$$

$$= \frac{1001}{2} \times 2000 = 1002001$$

Example : Find the sum of all natural no. lying between 1008 which are multiples of 5.

Solution : Here, first term,

S_n = 105 + 110 + ... + 995

a = 105

d = 5

n = 180 – 1 = 179, l = 995 (last term)

$$\therefore \; S_n = \frac{n}{2}\{a + l\}$$

$$= \frac{179}{2}\{105 + 995\}$$

$$= \frac{179}{2} \times 1100 = 179 \times 550 = 98450$$

Example : If the sum of certain number of terms of the A.P. 25, 22, 19, ... is 116. Find the last term.

Solution : Here

a = 25

d = –3

S_n = 116

Let, there are n terms in the A.P.

$$S_n = \frac{n}{2}\{2a + (n-1)d\}$$

$$\Rightarrow \; 116 = \frac{n}{2}\{2 \times 25 + (n-1)(-3)\}$$

$\Rightarrow$ $232 = 50n - 3n^2 + 3n$

$\Rightarrow$ $3n^2 - 53n + 232 = 0$

$\Rightarrow$ $3n^2 - 24n - 29n + 232 = 0$

$\Rightarrow$ $3n(n - 8) - 29(n - 8) = 0$

$\Rightarrow$ $(n - 8)(3n - 29) = 0$

$\Rightarrow$ $(n - 8)$ or $n = \frac{29}{3}$

$\therefore$ n = 8 $\qquad \because \; n \neq \frac{29}{3}$

T_8 = a + 7d = 25 + 7 × (–3) = 25 – 21 = 4

Example : For what value of n

$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$ is the A.M. of a and b?

Solution : We know that A.M. between a and b is $\frac{a+b}{2}$

$$\frac{a^{n+1}+b^{n+1}}{a^n+b^n}$$

$$\Rightarrow \quad 2a^{n+1}+2b^{n+1}=a^{n+1}+ab^n+a^nb+b^{n+1}$$

$$\Rightarrow \quad a^{n+1}+b^{n+1}=ab^n+a^nb$$

$$\Rightarrow \quad a^n(a-b)+b^n(b-a)=0$$

$$\Rightarrow \quad a^n-b^n=0$$

$$\Rightarrow \quad a^n=b^n$$

$$\Rightarrow \quad \frac{a^n}{b^n}=1 \quad \Rightarrow \left(\frac{a}{b}\right)^n=1=\left(\frac{a}{b}\right)^0 \quad \Rightarrow \text{ n = 0}$$

Example : In an A.P. if p[th] term is $\frac{1}{q}$ and q[th] term in $\frac{1}{p}$ .Prove that sum of first P, Q terms.

$\frac{1}{2}(pq+1)$ where $p \neq q$

Solution : Let, a be the first term and d be the common difference, then

$$S_p=\frac{p}{2}\{2a+(p-1)d\}$$

$$\Rightarrow \quad \frac{1}{q}=\frac{p}{2}\{2a+(p-1)d\} \qquad \text{.................. (1)}$$

$$=\frac{q}{2}\{2a+(q-1)d\}$$

$$\Rightarrow \quad \frac{1}{p}=\frac{q}{2}\{2a+(q-1)d\} \qquad \text{.................. (2)}$$

Now $\quad S_{pq}=\frac{pq}{2}\{2a+(pq-1)d\}$ .... (3)

(1) $\quad \Rightarrow \quad \frac{2}{pq}=2a+(p-1)d$

(2) $\Rightarrow \dfrac{2}{pq} = 2a + (q-1)d$

Subtracting we get,

$\{(p - 1) - (q - 1)\}$ d = 0

$\Rightarrow$ (p – q).d = 0 $\Rightarrow$ d

t_p = a + (p –1) d

$\dfrac{1}{q} = a(p-1)d$

$\Rightarrow$ 1 = aq + q (p – 1) d = 0 .... (1)

t_q = a + (q –1) d

$\dfrac{1}{p} = a(q-1)d$

$\Rightarrow$ 1 = ap + p(q – 1) d = 0 .... (2)

$S_{pq} = \dfrac{pq}{2}\{2a + (pq-1)d\}$ .... (3)

(1) × p $\Rightarrow$ p = apq + pq(p – 1) d = 0

(2) × q $\Rightarrow$ q = apq + pq(q – 1) d = 0

(–) $\Rightarrow$ p – q = pqd{p – 1 – q + 1}

$\Rightarrow$ (p – q) = pqd(p – q)

$\Rightarrow d = \dfrac{1}{pq}$

(1) $\Rightarrow 1 = aq + q(p-q)\dfrac{1}{pq}$

$\Rightarrow p = aqp + p - 1$

$\Rightarrow apq = 1 \Rightarrow a = \dfrac{1}{pq}$

(3) $\Rightarrow S_{pq} = \dfrac{pq}{2}\left\{\dfrac{2}{pq} + (pq-1)\dfrac{1}{pq}\right\}$

$= \dfrac{pq}{2}\left\{\dfrac{2}{pq} + 1 - \dfrac{1}{pq}\right\}$

$= \dfrac{pq}{2}\left\{\dfrac{1}{pq} + 1\right\}$

$= \dfrac{1}{2}\{pq + 1\}$

Example : Sum of the first p, q and r terms of an A.P. are a, b and c respectively. Prove that

$$\frac{a}{p}(q-r) + \frac{b}{q}(r-p) + \frac{c}{r}(p-q) = 0$$

Solution : Let, x be the first term and d be the c.d.

$$S_p = \frac{p}{2}\left\{2x+(p-1)d\right\}$$

$$\Rightarrow \quad a=\frac{p}{2}\left\{2x+(p-1)d\right\}$$

$S_q =$

$$\frac{q}{2}\left\{2x+(q-1)d\right\}$$

$\Rightarrow$

$$\therefore \quad c=\frac{r}{2}\left\{2x+(r-1)d\right\}$$

$$\therefore \quad \frac{a}{p}=x+\frac{1}{2}(p-1)d \qquad \text{.................. (1)}$$

$$\frac{b}{q}=x+\frac{1}{2}(q-1)d \qquad \text{.................. (2)}$$

$$\frac{c}{r}=x+\frac{1}{2}(r-1)d \qquad \text{.................. (3)}$$

(1) × (q – r) + (2) (r – p) + (3) × (p – q)

$$\Rightarrow \quad \frac{a}{p}(q-r)+\frac{b}{q}(r-p)+\frac{c}{r}(p-q)$$

$$=x\left\{q-r+r-p+p-q\right\}+\frac{d}{2}\left\{(p-1)(q-r)(q-1)(r-p)+(r-1)(p-q)\right\}$$

$$=x\times 0+\frac{d}{2}\left\{pq-pr-q+r+qr-qp-r+p+rp-rq-p+q\right\}$$

$$=x+\frac{d}{2}\times 0=0$$

Geometric Progression (G.P)

A sequence of number a_1, a_2, ..., a_n, ... is said to be in G.P. if each a_i is non zero and

$$\frac{a_2}{a_1}=\frac{a_3}{a_2}=\frac{a_4}{a_3}=.........-r\ (const)$$

Here, a_1 is called first term and generally it is denoted by a and r is called common ratio.

Now, $t_1 = a_1 = a$

$t_2 = a_2 = ar$

$t_3 = a_3 = ar^2$

$t_4 = ar^3$

....

....

$t_n = ar^{n-1}$

Example : If 2 is the first term and $\frac{1}{2}$ is the common ratio. Find the 10th term.

Solution : Here, $a = 2$

$$r = \frac{1}{2}$$

$$t_{10} = ar^{10-1}$$

$$= 2\times\left(\frac{1}{2}\right)^0$$

$$= \frac{1}{2^8} = \frac{1}{256}$$

Example : Which term of the following sequence $2, \sqrt{2}, 4$............... is 128?

Solution : Here, $a = 2$

$$r = \frac{2\sqrt{2}}{2} = \sqrt{2}$$

Let, 128 is the n^{th} term

$$t_n = ar^{n-1}$$

$$\Rightarrow \quad 128 = 2\times\left(\sqrt{2}\right)^{n-1}$$

$$\Rightarrow \quad 2^7 = 2\times 2^{\left(\frac{n-1}{2}\right)} = 2^{1+\frac{n-1}{2}}$$

$$\Rightarrow \quad 7 = \frac{2+n-1}{2}$$

$$\Rightarrow \quad 14 = n+1$$

$$\Rightarrow \quad n = 13$$

$\therefore$ 13^{th} term is 128.

Example : The 5^{th}, 8^{th} and 11^{th} terms of a G.P. are p, q and s respectively. S.T. $q^2 = ps$

Solution : Let, a be the first term and r be the common ratio.

$$t_5 = ar^{5-1}$$

$$\Rightarrow \quad p = ar^4 \quad \text{............. (1)}$$

$$t_8 = ar^{8-1}$$

$$\Rightarrow \quad q = ar^7 \quad \text{............. (2)}$$

$$t_{11} = ar^{11-1}$$

$$\Rightarrow \quad r = ar^{10} \quad \text{............. (3)}$$

Now, $q^2 = a^2r^{14}$

$$= \left(ar^4\right)\left(ar^{10}\right)$$

$$= P.S.$$

* Sum of n terms of a.G.P.

Let the G.P. be a, ar, ar^2, ..., ar^{n-1}

Let $S_n = a + ar + ar^2 + \ldots\ldots\ldots + ar^{n-1}$ .... (1)

$\Rightarrow$ $rs_n = ar + ar^2 + \ldots\ldots\ldots + ar^{n-1} + ar^n$ .... (2)

(2) – (1) $\Rightarrow$ $(r-1)s_n = ar^n - a = a(r^n - 1)$

$$\Rightarrow s_n = \frac{a(r^n-1)}{r-1}\,(r>1)$$

$$\text{Or } s_n = \frac{a(1-r^n)}{1-r}\,(r<1)$$

> *If* $r>1$
>
> $\lim\limits_{n\to\infty} S_n = \lim\limits_{n\to\infty} \frac{a(r^n-1)}{r-1} = \infty$

Example : Find the sum of the first 20th terms of geometric series.

0.15 + 0.015 + 0.0015 + ...

Solution : Here, a = 0.15

$$r = \frac{0.015}{0.15} = 0.1$$

$$S_{20} = \frac{a(1-r^n)}{1-r} = \frac{0.15\{1-(0.1)^{20}\}}{1-0.1}$$

$$= \frac{0.15}{.9}\left\{1-\left(\frac{1}{10}\right)^{20}\right\}$$

$$= \frac{15}{90}\left\{1-\frac{1}{10^{20}}\right\} = \frac{1}{6}\left\{1-\frac{1}{10^{20}}\right\}$$

Geometric mean : If a, b, c are in G.P. then b is called G.M. between a and c.

$$\frac{b}{a} = \frac{c}{b}$$

$$\Rightarrow b^2 = ac$$

Question : Insert five G.M. between 1 and 64.

Solution : Here, a =1

Let, r be the c.r.

Also, 64 is the 7th term

$t_r = ar^{7-1}$

$\Rightarrow$ $64 = 1.r^6$

$\Rightarrow$ $2^6 = r^6$

$\Rightarrow$ r = 2

$\therefore$ The G.M.S are 2, 4, 8, 16, 32 //

Relationship between A.M. and G.M.

Let, A and G be the A.M. and G.M. of two numbers a and b respectively.

$$\therefore A = \frac{a+b}{2},\ G = \sqrt{ab}$$

Now, $A - G = \frac{a+b}{2} - \sqrt{ab} = \frac{1}{2}\left(a+b-2\sqrt{ab}\right) = \frac{1}{2}\left(\sqrt{a}-\sqrt{b}\right)^2 \geq 0$

Exercise : Sum of first 3rd terms is $\frac{39}{10}$ their product is 1. Find the common ratio and the term.

Solution : Let, the three terms be $\frac{a}{r}, a, ar$

$\therefore \quad \frac{a}{r}+a+ar=\frac{39}{10}$

$\Rightarrow \quad a\left(\frac{1}{r}+1+r\right)=\frac{39}{10}$ .... (1)

$\therefore \quad \frac{a}{r}.a.ar=1$

$\Rightarrow \quad a^3=1 \quad \Rightarrow \quad a=1$

(1) $\Rightarrow \quad \frac{1}{r}+1+r=\frac{39}{10}$

$\Rightarrow \quad \frac{1+r+r^2}{r}=\frac{39}{10}$

$\Rightarrow \quad 10+10r+10r^2=39r$

$\Rightarrow \quad 10r^2-29r+10=0$

$\Rightarrow \quad 10r^2-25r-4r+10=0$

$\Rightarrow \quad 5r(2r-5)-2(2r-5)=0$

$\Rightarrow \quad (5r-2)(2r-5)=0$

$\therefore \quad 5r-2=0 \quad , \quad 2r-5=0$

$\Rightarrow \quad r=\frac{2}{5}$ Or $r=\frac{5}{2}$

When $r=\frac{5}{2}$, then terms are

$\frac{a}{r}=\frac{2}{5}, a=1, ar=\frac{5}{2}$

When $r=\frac{2}{5}$, the terms are

$\frac{a}{r}=\frac{5}{2}, a=1, ar=\frac{2}{5}$

Example : How many terms of G.P. 3, 3^2, 3^3, ... are needed to give the sum 120?

Solution : Here, a = 3, r = 3

Let, n be the number of terms

Now,

$$S_n=\frac{a\left(r^n-1\right)}{r-1}$$

$\Rightarrow \quad 120 = \dfrac{3(3^n - 1)}{3-1}$

$\Rightarrow \quad 80 = 3^n - 1$

$\Rightarrow \quad 3^n = 81 = 3^4$

$\Rightarrow \quad n = 4$

Example : The sum of first three terms of a G.P. is 16 and the sum of next three term is 128. Determine the first term, common ratio and sum of to n terms.

Solution : Let, a be the first term and
r be the common ratio

Then, $a + ar + ar^2 = 16$

$\Rightarrow \quad a(1 + r + r^2) = 16$ .... (1)

$ar^3 + ar^4 + ar^5 = 128$

$\Rightarrow \quad ar^3(1 + r + r^2) = 128$ .... (2)

(2) ÷ (3) $\Rightarrow \quad r^3 = \dfrac{128}{16} = 8 = 2^3$

$\therefore \quad r = 2$

(1) $\Rightarrow \quad a(1 + 2 + 2^2) = 16$

$\therefore \quad a = \dfrac{16}{7}$

$$S_n = \frac{a(r^n - 1)}{r-1} = \frac{\frac{16}{7}(2^n - 1)}{2-1} = \frac{16}{7}(2^n - 1)$$

Example : Find the sum to n terms of the sequence 8, 88, 888, 8888, ...

Solution : S_n = 8 + 88 + 888 + 8888 + ... to n terms
= 8(1 + 11 + 111 + 1111 + ... to n terms)

$$= \frac{8}{9}(9 + 99 + 999 + 9999 + \text{............to } n \text{ terms})$$

$$= \frac{8}{9}\left[(10-1) + (10^2 - 1) + (10^3 - 1) + (10^4 - 1) + \text{.........to } n \text{ terms}\right]$$

$$= \frac{8}{9}\left[(10 + 10^2 + 10^3 + 10^4 + \text{.........to } n \text{ terms}) - n\right]$$

$$= \frac{8}{9}\left[\frac{10(10^n - 1)}{10-1} - n\right]$$

$$= \frac{8}{9}\left[\frac{10}{9}(10^n - 1) - n\right]$$

Example : If the p^{th} and 9^{th} and r^{th} term of a G.P. are a, b and c respectively, then P.T.
$a^{q-r} \cdot b^{r-p} \cdot c^{p-q} = 1$

Solution : Let, x be the first term and y be the common ratio

$t_p = xy^{p-1} \Rightarrow a = xy^{p-1}$

Similarly, $b = xy^{q-1}$

$c = xy^{r-1}$

LHS $= a^{q-r} . b^{r-p} . c^{p-q}$

$= \left\{ \left(xy^{p-1}\right)^{q-r} . \left(xy^{q-1}\right)^{r-b} . \left(xy^{r-1}\right)^{p-r} \right\}$

$= x^{q-r} y^{(p-1)(q-r)} x^{r-p} y^{(q-1)(r-p)} x^{p-r} y^{(r-1)(p-r)}$

$= x^{q-r+r-p+p-q} y^{pq-pr-q+r+pr-qp-r+p+rp-rq-p+q}$

$= x^0 y^0 = 1 = RHS$

Example : If the first n[th] term of a G.P. are a and b respectively and p is the product of first n terms, then P.T.

$p^2 = (ab)^n$

Solution : Let, first term = a

r be the common ratio'

n[th] term $= ar^{n-1}$

$\Rightarrow b = ar^{n-1}$

$p = a. ar. ar^2 .... ar^{n-1}$

$= a^n r^{1+2+.................+(n-1)}$

$= a^n r^{\frac{(n-1)n}{2}}$

$p^2 = a^{2n} a^{(n-1)n} = \left(a^2 r^{n-1}\right)^n = (ab)^n$ using (1)

Again, $ab = a.ar^{n-1} = a^2 r^{n-1}$ .... (1)

Example : If S be the sum, p the product & R the sum of retiprocal of n terms in a G.P. then P.T. $P^2 R^n = S^n$

Solution : Let, a be he first term

r be the common ratio

Then, the G.P. is

$a, ar, ar^2, ...$

A/Q,

S $= a + ar + ar^2 + ar^3 + ... + ar^{n-1}$

$= \frac{a\left(r^n - 1\right)}{r - 1}$ .... (1)

P $= a.ar. ar^2 ....ar^{n-1}$

$= a^n . r^{1+2+........+(n-1)} = a^n . r^{\frac{(n-1)n}{2}}$ .... (2)

$$R = \frac{1}{a} + \frac{1}{ar} + \frac{1}{ar^2} + \ldots\ldots\ldots\ldots\ldots + \frac{1}{ar^{n-1}}$$

$$= \frac{r^{n-1} + r^{n-2} + r^{n-3} + \ldots\ldots\ldots\ldots + 1}{ar^{n-1}}$$

$$= \frac{1.\left(\frac{r^n - 1}{r - 1}\right)}{a.r^{n-1}} = \frac{r^n - 1}{a(r-1)\left(r^{n-1}\right)} \qquad \ldots\ldots\ldots\ldots (3)$$

Now, $$P^2 R^n = \left(a^n\ r^{\frac{(n-1)n}{2}}\right)^2 \left\{\frac{r^n - 1}{a(r-1)\left(r^{n-1}\right)}\right\}^n$$

$$= a^{2n}.r^{(n-1)n}\ \frac{\left(r^n - 1\right)^n}{a^n (r-1)^n\ r^{(n-1)n}}$$

$$= a^n\ \frac{\left(r^n - 1\right)^n}{(r-1)^n} = S^n \quad \text{[using (1)]}$$

Example : Find the value of n so that $\frac{a^{n+1} + b^{n+1}}{a^n + b^n}$ may be the G.M. between a and b.

Solution : We know that, G.M. between a and b is $\sqrt{ab}$

$$\therefore \quad \frac{a^{n+1} + b^{n+1}}{a^n + b^n} = \sqrt{ab}$$

$$\Rightarrow \quad a^{n+1} + b^{n+1} = \sqrt{ab}\left(a^n + b^n\right)$$

$$= b^{\frac{1}{2}} + a^{n+\frac{1}{2}} + b^{n+\frac{1}{2}} + a^{\frac{1}{2}}$$

$$\Rightarrow \quad a^{n+1} - b^{\frac{1}{2}} a^{n+\frac{1}{2}} = b^{n+\frac{1}{2}} a^{\frac{1}{2}} - b^{n+1}$$

$$\Rightarrow \quad a^n\left(a - b^{\frac{1}{2}} a^{\frac{1}{2}}\right) = b^n\left(b^{\frac{1}{2}} a^{\frac{1}{2}} - b\right)$$

$$\Rightarrow \quad a^n \sqrt{a}\left(\sqrt{a} - \sqrt{b}\right) = b^n \sqrt{b}\left(\sqrt{a} - \sqrt{b}\right)$$

$$\Rightarrow \quad a^n \sqrt{a} = b^n \sqrt{b}$$

$$\Rightarrow \quad \frac{a^n}{b^n} = \frac{\sqrt{b}}{\sqrt{a}}$$

$$\Rightarrow \left(\frac{a}{b}\right)^n = \left(\frac{b}{a}\right)^{\frac{1}{2}} = \left(\frac{a}{b}\right)^{-\frac{1}{2}}$$

$$\Rightarrow n = -\frac{1}{2}$$

Example : The sum of two nos. is 6 times of their G.M. So that the numbers are in the ratio $\left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$

Solution : Let, the numbers be a and b

$$a+b = 6\sqrt{ab}$$

$$\Rightarrow \frac{a}{b}+1 = 6\sqrt{\frac{a}{b}} \quad (\div b)$$

$$\Rightarrow x^2+1 = 6x \quad \left[if\ x = \sqrt{\frac{a}{b}}\right]$$

$$\Rightarrow x^2-6x+1 = 0$$

$$\therefore x = \frac{-(-6)+\sqrt{(-6)^2-4.1.1}}{2}$$

$$= \frac{6\pm\sqrt{32}}{2}$$

$$= \frac{6\pm 4\sqrt{2}}{2} = 3\pm 2\sqrt{2}$$

$$\Rightarrow \sqrt{\frac{a}{b}} = 3\pm 2\sqrt{2}$$

Let, $\sqrt{\frac{a}{b}} = 3+2\sqrt{2}$

$$\Rightarrow \frac{a}{b} = \left(3+2\sqrt{2}\right)^2$$

$$= \frac{\left(3+2\sqrt{2}\right)\left(3+2\sqrt{2}\right)\left(3-2\sqrt{2}\right)}{\left(3-2\sqrt{2}\right)}$$

$$= \frac{\left(3+2\sqrt{2}\right)(9-8)\,3+2\sqrt{2}}{\left(3-2\sqrt{2}\right)} = \frac{3+2\sqrt{2}}{3-2\sqrt{2}}$$

$$\therefore a:b = \left(3+2\sqrt{2}\right):\left(3-2\sqrt{2}\right)$$

Example : If A and G be A.M. and G.M. respectively between two positive numbrs, P.T. the numbers are $A\pm\sqrt{(A+G)(A-G)}$

Solution : Let, a and b be the two numbers

then $\frac{a+b}{2} = A,\ \sqrt{ab} = G$

$\Rightarrow \quad a+b = 2A,\ \Rightarrow ab = G^2$

If a and b are the roots of a quadratic equation then the equation

$$x^2 - (a+b)x + ab = 0$$

$$\Rightarrow \quad x^2 - 2Ax + G^2 = 0$$

$$\Rightarrow \quad x = \frac{2A \pm \sqrt{4A^2 - 4G^2}}{2.1}$$

$$= \frac{2A \pm 2\sqrt{A^2 - G^2}}{2}$$

$$= A \pm \sqrt{(A+G)(A-G)}$$

$\therefore$ The roots of the equation are

$$A \pm \sqrt{(A+G)(A-G)}$$

i.e. the values of a and b are given by

$$A \pm \sqrt{(A+G)(A-G)}$$

Example : The ratio of A.M. and G.M. of two positive numbers a and b is m : n, then so that $a:b = \left(m + \sqrt{m^2 - n^2}\right) : \left(m - \sqrt{m^2 - n^2}\right)$

Solution : We know that, A.M. and G.M. of 2 numbers a and b are $\frac{a+b}{2}$ and $\sqrt{ab}$ respectively.

$$\therefore \quad \frac{\frac{a+b}{2}}{\sqrt{ab}} = \frac{m}{n}$$

$$\Rightarrow \quad \frac{a+b}{2\sqrt{ab}} = \frac{m}{n} \quad \text{.............. (1)}$$

$$\Rightarrow \quad \frac{(a+b)^2}{4ab} = \frac{m^2}{n^2}$$

$$\Rightarrow \quad \frac{(a+b)^2 - 4ab}{4ab} = \frac{m^2 - n^2}{n^2}$$

$$\Rightarrow \quad \frac{(a-b)^2}{4ab} = \frac{m^2 - n^2}{n^2}$$

$$\Rightarrow \quad \frac{a-b}{2\sqrt{ab}} = \frac{\sqrt{m^2 - n^2}}{n} \quad \text{.............. (2)}$$

(1) + (2) $\Rightarrow$ $\frac{2a}{2\sqrt{ab}} = \frac{m+\sqrt{m^2-n^2}}{n}$ .... (3)

(1) – (2) $\Rightarrow$ $\frac{2b}{2\sqrt{ab}} = \frac{m-\sqrt{m^2-n^2}}{n}$ .... (4)

(3) ÷ (4) $\Rightarrow$ $\frac{a}{b} = \frac{m-\sqrt{m^2-n^2}}{m-\sqrt{m^2-n^2}}$

Sum to n terms of special series :

1. 1 + 2 + 3+ 4 + 5 + ...

Solution :

S_n = 1 + 2 + 3 + ...+ n

$= \frac{n}{2}\{2\times 1 + (n-1)\times 1\}$ [a = 1, d = 1]

$= \frac{n}{2}\{2 + n - 1\}$

$= \frac{n(n+1)}{2}$

2. $1^2 + 2^2 + 3^2 + 4^2 + ...$

Solution : Let,

S_n = $1^2 + 2^2 + 3^2 + ... + n^2$

We have, $r^3 - (r-1)^3 = r^3 - (r^3 - 3r^2 + 3r - 1) = 3r^2 - 3r + 1$

Putting r = 1, 2, 3, ... n we have

$1^3 - 0^3 = 3.1^2 - 3.1 + 1$

$2^3 - 1^3 = 3.2^2 - 3.2 + 1$

$3^3 - 2^3 = 3.3^2 - 3.3 + 1$

....

$n^3 - (n-1)^3 = 3.n^2 - 3n + 1$

Adding we have

$n^3 = 3(1^2 + 2^2 + 3^2 + ... + n^2) - 3(1 + 2 + 3......... + n) + n$

$\Rightarrow$ $n^3 = 3S_n - 3\frac{n(n+1)}{2} + n$

$\Rightarrow$ $S_n = n^3 + \frac{3n(n+1)}{2} - n$

$= \frac{2n^3 + 3n^2 + 3n - 2n}{2}$

$= \frac{2n^3 + 3n^2 + n}{2}$

$$= \frac{n\left(2n^2 + 3n + 1\right)}{2}$$

$$= \frac{n\left(2n^2 + 2n + n + 1\right)}{2}$$

$$\Rightarrow \quad S_n = \frac{n\{2n(n+1) + (n+1)\}}{6} = \frac{n(n+1)(2n+1)}{6}$$

3. $1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots\ldots + n^3$

Solution : Let,

$S_n = 1^3 + 2^3 + 3^3 + \ldots\ldots\ldots\ldots\ldots + n^3$

We have, $r^4 - (r-1)^4 = r^4 - r^4 + 4r^3 - 6r^2 + 4r - 1$

$$= 4r^3 - 6r^2 + 4r - 1$$

Putting r = 1, 2, 3, ... n

We have ,

$$1^4 - 0^4 = 4\times 1^3 - 6\times 1^2 + 4\times 1 - 1$$

$$2^4 - 1^4 = 4\times 2^3 - 6\times 2^2 + 4\times 2 - 1$$

$$3^4 - 2^4 = 4\times 3^3 - 6\times 3^2 + 4\times 3 - 1$$

....

$$n^4 - (n-1)^4 = 4n^3 - 6n^2 + 4n - 1$$

Adding we have

$$n^4 = 4(1^3 + 2^3 + 3^3 + \ldots\ldots + n^3) - 6(1^2 + 2^2 + 3^2 + \ldots\ldots + n^2) + 4(1 + 2 + 3 + \ldots\ldots + n)$$

$$\Rightarrow \quad n^4 = 4S_n - 6\frac{n(n+1)(2n+1)}{6} + 4\frac{n(n+1)}{2} - n$$

$$\Rightarrow \quad 4S_n = n^4 + n(n+1)(2n+1) - 2n(n+1) + n$$

$$= n\left\{n^3 + 2n^2 + n + 2n + 1 - 2n - 2 + 1\right\}$$

$$= n^2\left\{n^2 + 2n + 1\right\} = n^2(n+1)^2$$

$$\therefore \quad S_n = \frac{n^2(n+1)^2}{4} = \left\{\frac{n(n+1)}{2}\right\}^2$$

Example-1 : Find the sum to n terms of the series

3 + 7 + 14 + 24 + 37 + ...

Solution :

Let, $S_n = 3 + 7 + 14 + 24 + 37 + \ldots\ldots\ldots\ldots\ldots + a_{n-1} + a_n$ ......(1)

Again, $S_n = 3 + 7 + 14 + \ldots\ldots\ldots\ldots\ldots + a_{n-2} + a_{n-1} + a_n$ ......(2) [Where a_{n-1} is $(n-1)^{th}$ term a_n is n^{th} term]

(1) – (2) $\Rightarrow$ $0 = 3 + \left(4 + 7 + 10 + 13 + \ldots\ldots\ldots\ldots\ldots + r_{n-2} + t_{n-1}\right) - a_n$

$$\Rightarrow a_n = 3 + \left(4 + 7 + 10 + 13 + \ldots\ldots\ldots\ldots\ldots + t_{n-1}\right)$$

$$= 3 + \frac{n-1}{2}\left\{2\times 4(n-1)\times 3\right\} \qquad [t_{n-1} \text{ is } (n-1)^{th} \text{ term}]$$

$$= 3 + \frac{n-1}{2}\left\{8 + 3n - 6\right\} \qquad [a = 4,\ d = 3,\ n \rightarrow n-1]$$

$$= 3 + \frac{n-1}{2}(3n+2)$$

$$= \frac{6 + (n-1)\,(2+3n)}{2}$$

$$= \frac{6 + 2n + 3n^2 - 2 - 3n}{2} = \frac{3n^2 - n + 4}{2}$$

$$\Rightarrow \quad a_n = \frac{1}{2}\left(3n^2 - n + 4\right)$$

$$\sum_{n=1}^{n} a_n = \frac{1}{2}\left[3\sum_{n=1}^{n} n^2 - \sum_{n=1}^{n} n + \sum_{n=1}^{n} 4\right]$$

$$\Rightarrow \quad S_n = \frac{1}{2}\left[3\times\frac{n(n+1)\,(2n+1)}{6} - \frac{n(n+1)}{2} + 4n\right]$$

$$= \frac{1}{2}\left[\frac{n(n+1)\,(2n+1)}{2} - \frac{n(n+1)}{2} + 4n\right]$$

$$= \frac{n}{2}\left[\frac{2n^2 + n + 2n + 1 - n - 1 + 8}{2}\right]$$

$$= \frac{n}{2}\left[2n^2 + 2n + 8\right]$$

$$= \frac{n}{2}\left[n^2 + n + 4\right]$$

Example-2 : $1\times 2 + 2\times 3 + 3\times 4 + 4\times 5 + \ldots\ldots\ldots\ldots\ldots$.

Solution : Here, n^{th} term : $t_n = n(n+1) = n^2 + n$

$$\sum_{n=1}^{n} t_n = \sum_{n=1}^{n} n^2 + \sum_{n=1}^{n} n$$

$$\Rightarrow \quad 1\times 2 + 2\times 3 + 3\times 4 + 4\times 5 + \ldots\ldots\ldots n(n+1) = \frac{n(n+1)(2n+1)}{6} + \frac{(n+1)}{2}$$

$$= \frac{n(n+1)(2n+1) + 3n(n+1)}{6}$$

$$= \frac{n(n+1)\left\{2n + 3 + 1\right\}}{6}$$

$$= \frac{n(n+1)\, 2(n+2)}{6} = \frac{n(n+1)\,(n+2)}{3}$$

Example-3 : $1\times 2\times 3 + 2\times 3\times 4 + 3\times 4\times 5 + ...$

Solution : Here, n^th^ term : $t_n = n(n+1)(n+2) = \left(n^2+n\right)\left(n+n\right) = n^3 + 3n^2 + 2n$

Putting n = 1, 2, 3, ..., n we have

$$t_1 = 1^3 + 3\times 1^2 + 2\times 1$$

$$t_2 = 2^3 + 3\times 2^2 + 2\times 2$$

$$t_3 = 3^3 + 3\times 3^2 + 2\times 3$$

....

$$t_n = n^3 + 3\times n^2 + 2\times n$$

Adding we have,

$$t_1 + t_2 + t_3 + ... + t_n = \left(1^3 + 2^3 + 3^3 + ... + n^3\right) + 3\left(1^2 + 2^2 + ... + n^2\right) + 2\left(1 + 2 + 3 + ... + n\right)$$

$$= \left\{\frac{n(n+1)}{2}\right\}^2 + 3.\frac{n(n+1)(2n+1)}{6} + 2.\frac{n(n+1)}{6}$$

$$= \frac{n^2(n+1)^2}{4} + \frac{n(n+1)(2n+1)}{2} + n(n+1)$$

$$= n(n+1)\left\{\frac{n(n+1)}{4} + \frac{(2n+1)}{2} + 1\right\}$$

$$= n(n+1)\left\{\frac{n(n+1) + 4n + 2 + 4}{4}\right\}$$

$$= n(n+1)\left\{\frac{n^2 + n + 2n^2 + 6}{4}\right\}$$

$$= \frac{n(n+1)}{4}\left(n^2 + 2n + 3n + 6\right)$$

$$= \frac{n(n+1)}{4}\left\{n\left(n+2\right) + 3\left(n+2\right)\right\}$$

$$= \frac{n(n+1)\left(n+2\right)\left(n+3\right)}{4}$$

Example-4 : $\frac{1}{1\times 2} + \frac{1}{2\times 3} + \frac{1}{3\times 4} + ...$

Solution : Here, $t_n = \frac{1}{n(n+1)} + \frac{1}{n} - \frac{1}{n+1}$

Putting n = 1, 2, 3, ..., n we have

$$t_1 = \frac{1}{1} - \frac{1}{11}$$

$$t_2 = \frac{1}{2} - \frac{1}{2+1}$$

$$t_3 = \frac{1}{3} - \frac{1}{3+1}$$

....

$$t_n = \frac{1}{n} - \frac{1}{n+1}$$

Adding we get

$$t_1 + t_2 + t_3 + ... + t_n = 1 - \frac{1}{n+1} = \frac{n+1-1}{n+1}$$

$$\Rightarrow \quad \frac{1}{1\times 2} + \frac{1}{2\times 3} + ... + \frac{1}{n(n+1)} = \frac{n}{n+1}$$

Example-5 : $5^2 + 6^2 + 7^2 + ... + 20^2$

Solution : We have,

Putting n = 1, 2, 3, ..., n we have

$$\left(1^2 + 2^2 + 3^2 + 4^2\right) + \left(5^2 + 6^2 + 7^2 + ... + 20^2\right) = \frac{20(20+1)\,(2\times 20+1)}{6}$$

$$\Rightarrow \quad 30 + \left(5^2 + 6^2 + 7^2 + ... + 20^2\right) = \frac{20\times 21\times 41}{6} = 2870$$

$$\Rightarrow \quad 5^2 + 6^2 + 7^2 + ... + 20^2 = 2840$$

Example-6 : $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + ...$

Solution : $t_n = \frac{1^2 + 2^2 + 3^3 + ... + n^3}{1+3+5+........... + (2n-1)}$ [a = 1, d = 2, n = n

$$= \frac{\left\{\frac{n(n+1)}{2}\right\}^2}{4n^2} \qquad \left[S_n = \frac{n}{2}\{2\times 1(n-1)\times 1\} = \frac{n}{2}(2n) = n^2\right.$$

$$= \frac{n^2(n+1)^2}{4n^2} = \frac{1}{4}\left[n^2 + 2n + 1\right]$$

$$\sum_{n=1}^{n} t_n = \frac{1}{4}\left[\sum_{n=1}^{n} n^2 + 2\sum_{n=1}^{n} n + \sum_{n=1}^{n} 1\right]$$

$$=\frac{1}{4}\left[\frac{n(n+1)(2n+1)}{6}+2.\frac{n(n+1)}{2}+n\right]$$

$$=\frac{n}{4}\left[\frac{2n^2+n+2n+1+6n+6+6}{6}\right]$$

$$=\frac{n}{4}\left[\frac{2n^2+9n+13}{6}\right] = \frac{n(2n^2+9n+13)}{24}$$

Example-7 : Find the sum of integers from a to 100 that are divisible by 2 or 5

Solution : Required sum = (2 + 4 + 6 + 8 + 10+ .... + 100) + (5 + 10 + 15 + ... + 100) – (10 + 20 + 30 + 40+ .... + 100)

$$=\frac{50}{2}\{2\times 2+(50-1)\times 2\}+\frac{20}{2}\{2\times 5+(20-1)\times 5\}-\frac{10}{2}\{2\times 10+(10-1)\times 10\}$$

$$=25\{4+98\}+10\{10+95\}-5\{20+90\}$$

= 2550 + 1050 – 550

$$=3600-550=3015$$

Example-1 : If $a^{\frac{1}{x}}=b^{\frac{1}{y}}=c^{\frac{1}{z}}$ and a, b, c, are in G.P. then P.T. x, y, z are in A.P.

Example-2 : Find four numbers in G.P. whose sum is 85 and product is 4096

Example-3 : If $\frac{1}{q+r},\frac{1}{r-p},\frac{1}{p+q}$ are in A.P. Then P.T. p^2,q^2,r^2 are in A.P

Example-4 : If the sum of three numbers in A.P. is 24 and their product is 440, find the numbers.

Example-5 : Find the sum to n terms whose k^{th} term is given by

(i) k(k + 1) (k + 4) (ii) k^2+2 (iii) $(2k-1)^2$

Harmonic Progression H.P.

a, b, c are in H.P. if $\frac{1}{a},\frac{1}{b},\frac{1}{c}$ are in A.P.

H.M. a, b, c in H.P.

$\Rightarrow \frac{1}{a},\frac{1}{b},\frac{1}{c}$ in A.P.

$\Rightarrow \frac{1}{b}-\frac{1}{a}=\frac{1}{c}-\frac{1}{b}$ $\qquad [AH\geq G^2]$

$\Rightarrow \frac{2}{b}=\frac{1}{a}+\frac{1}{c}=\frac{a+c}{ac}$

$\Rightarrow b=\frac{2ac}{a+c}\rightarrow$ Harmonic mean.

EXERCISE : 9

1. Find the first four terms of the sequence defined by

   (i) $a_n = 4n^2 + 3$ (ii) $a_n = n^{th}$ prime number

2. Write the next term in each of the following sequences

(i) 0, 2, 6, 12, 20 ....

(ii) 1, 5, 14, 30, 55 ....

3. First term of a sequence is 1 and the $(n+1)^{th}$ term is obtained by adding (n + 1) to the n^{th} term for all natural number n. Find the sixth term of the sequence.

4. If the third term of an A. P is 12 and the seventh term is 24, then find the 10^{th} term.

5. If a, b, c are in A. P then find the value of $\frac{a-b}{b-c}$

6. If a, b, c are the p^{th}, q^{th} and r^{th} terms respectively of an A.P, prove that

   (i) p (b –c) + q (c – a) + r (a – b) = 0

   (ii) a (q – r) + b (r – p) + c (p – q) = 0

7. Find the sum of first 19 terms of an A.P, whose n^{th} term is 2n + 1.

8. Find the sum of all natural numbers lying between 200 and 400 which are divisible by 7.

9. The ratio of the first A.M and the last A.M. of an arithmetic means between 5 and 35 is 1 : 4. Show that n = 9

10. Insert 12 arithmetic means between 4 and 43.

11. Arithmetic means are inserted between 7 and 71 in such a way that the 5^{th} A.M. is 27. Find the number of A.Ms inserted.

12. The sum of n terms of an A.P. is $pn + qn^2$, where p, q are constants. What is the common difference?

13. If the p^{th} and the q^{th} terms of an A.P are a and b respectively, find the sum of the first (p + q) terms.

14. The first term of a G.P. is 27 and 8^{th} term is $\frac{1}{81}$. Find the sum of first 10 terms of the G.P.

15. The fourth term of a G.P. is the square of it's second term and the first term is –3. Determine the 7^{th} term.

16. If p^{th}, q^{th} and r^{th} term of a G. P are themselves in G.P, show that p, q, r are in A.P.

17. A G.P consists of an even number of terms. If the sum of all the terms is 5 times the sum of terms occupying odd places, then find the common ratio.

18. In a G.P the first term is 7, the last term is 448 and the sum is 889. Find the common ratio.

19. Find the sum to n terms

    0.5 + 0.55 + 0.555 + ...

20. If G_1 and G_2 are the two geometric means between 'a' and 'b', show that

$$\frac{G_1^{\,2}}{G_2} + \frac{G_2^{\,2}}{G_1} = a + b$$

21. a, b, c are in G.P, x is the A.M. between a and b, y is the A.M. between b and c, then show that

(i) $\frac{a}{x}+\frac{c}{y}=2$ (ii) $\frac{1}{x}+\frac{1}{y}=\frac{2}{b}$

22. Insert 4 G.M between $\frac{4}{9}$ and $\frac{27}{8}$.

23. If $\frac{1}{2y}$ is the A.M. between $\frac{1}{y-x}$ and $\frac{1}{y-z}$, prove that y is the G.M. between x and z.

24. The A.M. and G.M between two positive numbers are 10 and 8 respectively. Find the numbers.

25. Find the 20^{th} term of the series
$2 \times 4 + 4 \times 6 + 6 \times 8 +$ .... to n terms

26. If 1 + 2 + 3 + ... + n = 28, then find the value of $1^2 + 2^2 + 3^2 + \ldots + n^2$.

27. Find the sum of n terms of the series whose n^{th} term is n(n + 3).

28. Find the sum of squares of first 100 odd natural numbers.

29. Find the sum to n terms the series 5 + 11 + 19 + 29 + 41 + ...

30. Find the sum to n terms the series $1 \times 3 \times 5 + 2 \times 4 \times 6 + 3 \times 5 \times 7 +$ ....

31. Find the sum to n terms the series $1+\left(1+\frac{1}{2}\right)+\left(1+\frac{1}{2}+\frac{1}{4}\right)+$..................

32. If S_1, S_2, S_3 are the sums of first n natural numbers, their squares and their cubes respectively, show that $9S_2^2 = S_3(1 + 8S_1)$

ANSWERS

1. (i) 7, 19, 39, 67
   (ii) 2, 3, 5, 7
2. (i) 30, (ii) 91. [Hint : $5 = 1 + 2^2$, $14 = 5 + 3^2$, $30 = 14 + 4^2$,.............]
3. 21

4. 33

5. 1

7. 399

8. 8729

10. 7, 10, 13, ... 40

11. 15

12. 2q

13. $\frac{1}{2}(p+q)\left(a+b+\frac{a-b}{p-q}\right)$

14. $\frac{81}{2}\left(1-\frac{1}{3^{10}}\right)$

15. –2187

17. 4

18. 2

19. $\frac{5}{81}\left(9n-1+\frac{1}{10^n}\right)$

22. $\frac{2}{3}, 1, \frac{3}{2}, \frac{9}{4}$

24. (4, 16), (16, 4)

25. 1680

26. 140

27. $\frac{1}{3}n(n+1)(n+5)$

28. 1333300

29. $\frac{1}{3}n(n+2)(n+4)$

30. $\frac{1}{4}n(n+2)(n+4)(n+5)$

31. $2n-2+\frac{1}{2^{n-1}}$

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